A COMPARATIVE INVESTIGATION

OF

RIGID FRAME CONSTRUCTION

AND

HIPPED PLATE CONSTRUCTION

IN REINFORCED CONCRETE

A THESIS

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CHAPTER I

INTRODUCTION

Man's progress through the ages has been in direct proportion to the skill and capabilities of his engineering profession. The engineers, through research and application of basic scientific principles, are constantly striving to improve the economic status of all mankind. In former times this progress was restricted to the trial and error procedure, which eventually produced the codes of practice that exist today in the form of building codes. Today, the engineer has a more thorough understanding of the materials with which he works, and can verify his judgment through the application of mathematics and scientific principles. This thesis contains a description of some recent developments in the field of reinforced concrete and gives a comparison of their application to modern day construction.

The Problem

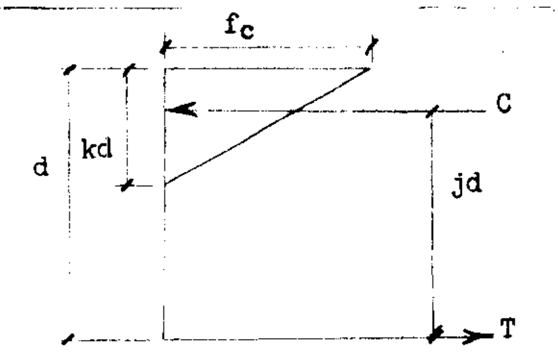
Statement of the Problem.--It was the purpose of this study (1) to design a given structure using rigid frame construction by the Elastic, Plastic and Ultimate design theories of Reinforced Concrete and compare the results with a design made using hipped plate construction; (2) to present the basic theory and design methods of the Plastic and Ultimate theories and hipped plate construction in one publication.

History of the Problem.--Concrete in the unreinforced state has been in use since the time of the early Romans. They used large chunks of rock as the aggregate and built many arches, aqueducts, walls and roads, some of which still exist. Reinforced concrete, as it is used today, was developed in France in 1850. A French gardener, Joseph Monier, built tanks and tubs of reinforced concrete, using concrete surrounding a wire framework. In 1850, another Frenchman, Lambot, built a small boat of reinforced concrete. Lambot took out some patents on the new material, believing it well adapted to shipbuilding and reservoir work. In 1861, Frances Coignet published his principles for the new construction, including applications to reinforced concrete beams, arches, pipes, etc. Both he and Monier performed some work for the Paris Exposition of 1867.[1]

The Elastic Theory, developed by Navier and Bernoulli, is the basis of the conventional design methods used today. It assumes, among other things, a homogeneous, isotropic material (which is not true for reinforced concrete) and that plane sections remain plane after bending. As a structural material, concrete is excellent in its ability to resist compression but is relatively weak in tension. Steel, on the other hand, is very strong in both tension and compression. However, steel is today (1953) more expensive and more difficult to obtain than concrete. The use of the two materials together affords a very efficient structural material as only a very small percentage of steel is required for the average structural section. As applied to reinforced

[1]Turneau, F. E. and E. R. Maurer, Principles of Reinforced Concrete Construction, 4th ed. New York, John Wiley and Sons, 1935.

concrete, the Elastic Theory assumes that the strains and therefore the stresses, are proportional to the distance from the neutral axis (the level at which the strains in the material are non-existent). The compressive stress distribution is assumed to be triangular in nature and its resultant force, C acts at the center of gravity of this



Assumed Stress Distribution for Bending

Figure 1

area (Figure 1). The tensile force, T, acts at the center of gravity of the steel reinforcement, and is equal to the steel area, A_s, times the allowable steel stress, f_s. The section dimensions are denoted as "b" for the width and "d" for the effective depth. By statics, since the sum of the horizontal forces must equal zero, the two resultants, C and T, are seen to be equal. The internal moments, produced by "C" and "T", must not be exceeded by bending moments produced by externally applied forces if the section is to remain in equilibrium. The strength of concrete varies according to the proportions of sand, cement, aggregate and water used in the mixing process. Reinforcing steels are manufactured with a fairly constant strength. As a result the dimensions of the triangular compressive area will vary for different grades of concrete. The elastic modulus, n which is the ratio of the modulus of elasticity of the steel, E_s, to that of the concrete, E_c, governs the design of reinforced concrete using the Elastic Theory. The value, n, is commonly taken as a round number (10, 12 or 15), because the elasticity modulus of concrete is difficult to determine and will vary for different batches of the same mixture. The constants "k" and "j" are determined from the elastic modulus, n, and the percentage of steel in the section, p, where

$$k = \quad 2np + (np)^2 - np \tag{1}$$

and

$$j = 1 - \frac{k}{3} \tag{2}$$

Design by the Elastic Theory is made rather easy by use of another design constant, K, which is derived from the internal resisting moments of the concrete and steel. The value, K, is equal to

$$K = \frac{M}{bd^2} = f_s \ p \ j = f'c \ k \ j \tag{3}$$

The value, K, provides a convenient means of determining the section dimensions required for a given external moment, M.

While the methods of reinforced concrete design have been somewhat modified and expanded since their original development, the basic theory has remained relatively unchanged. Concrete has been investigated and experimented with in the laboratory and today it is possible to evaluate the effects of temperature, shrinkage and plastic flow with an acceptable degree of accuracy. Laboratory tests have also revealed, however, that reinforced concrete structural members possessed an ultimate strength above that shown by the Elastic Theory.

Subsequent developments have shown that concrete possesses a distinct plastic strength under loading in addition to its elastic strength on which the existing design theory is based. Two new design theories have been developed in the last fifteen years which consider the plastic strength of the materials used. These theories have remained rather dormant during the past decade, but have recently been revived and placed in their proper perspective. At present, they are being used to determine the ultimate strength of reinforced concrete structures with respect to earthquake and blast resistance.

In 1940, Charles S. Whitney, a Milwaukee consulting engineer, published his Plastic Theory of Reinforced Concrete Design.[2] It is an empirically developed theory based on observations of results of reinforced concrete structural members tested to failure. Mr. Whitney determined that, for all practical purposes, the compressive stress distribution at failure may be represented by an equivalent rectangular block of stress. Even though the ultimate compressive stress distribution is parabolic in nature, this method produces a stress representation which closely approximates the area, magnitude and location of the actual forces. The Plastic Theory eliminates the use of the elastic modulus, n, and the method of application to reinforced concrete design is simple and direct. Mr. Whitney extended his method to apply to beams in flexure, compression, combined bending and axial loading,

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[2]Whitney, Charles S., "Plastic Theory of Reinforced Concrete Design", Transactions, American Society of Civil Engineers, 1940, vol. 66, pps. 1749-1780.

and column sections.

The second theory entitled, The Ultimate Theory of Reinforced Concrete Design, was published by Vernon P. Jensen of the University of Illinois, in 1943.[3] Mr. Jensen extended the present conventional theory to cover the plastic range also. The procedure introduces the plasticity ratio of concrete which was determined by Mr. Jensen to be a function of the compressive strength. He based his design theory on idealized stress-strain curves for concrete and steel, and found that the currently used modular ratio, "n", was applicable to the elastic range of concrete but that the plastic range was characterized by the plasticity ratio, B. The compressive stress distribution at failure is assumed to be trapezoidal in nature. The ultimate loading on a reinforced concrete structural member tested to failure compares very favorably with the results predicted by the Ultimate design theory.

As American designers have frequently turned to Europe for new approaches to the problem of material conservation a design of the given structure using hipped plate construction was included in this investigation. This type of construction was originated in Germany in 1925, and has been used extensively on the Continent since that time. A crude example of this type of structure would be an ordinary cardboard or wooden box, with the top removed, placed upside down on the ground. Such a structure would deflect considerably under loadings applied to its top, or roof. However, if the roof of the structure was made up of inclined slabs approximating a semi-circle, or in a saw-tooth arrangement,

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[3]Jensen, Vernon P., "Ultimate Design of Reinforced Concrete", Journal of American Concrete Institute, 1943, Vol. 14, p. 565.

considerable loadings could be supported without producing an appreciable distortion in the structure. Hipped plate construction was introduced to this country in 1936, when a warehouse with a saw-tooth type roof was erected on the West Coast. A number of similar type structures have been constructed since that time but they are, in general, confined to the West Coast region. The analysis of a hipped plate structure entails a procedure unlike that encountered for conventional construction. However, the design principles involved are those used in standard reinforced concrete design procedure and the analysis of a hipped plate structure is no more difficult than the average rigid frame design. Hipped plate structures are extensively used in Europe and Russia for bins, bunkers, hoppers and industrial structures.

Purpose of the Investigation

It was the purpose of this investigation (1) to make comparative designs for a given structure employing the Conventional, Plastic and Ultimate theories of reinforced concrete design plus a design as a hipped plate structure; (2) to present the basic principles of the newer design theories and hipped plate construction in one publication. The results of this investigation should, therefore, present an indication of the relative economic merits of each design method for the structure selected. As all engineering advances through the ages have been primarily economic in nature, this study should also provide a means for determining the feasibility of adopting new design standards for reinforced concrete construction.

Review of the Literature

All standard textbooks[4] on reinforced concrete design present the Conventional Design Theory and its applications in detail. The Portland Cement Association, a research and promotional organization sponsored by cement manufacturers, has made many new developments in reinforced concrete design and consturction available to practicing engineers through the distribution of informative literature.

The Plastic Theory of Reinforced Concrete Design, developed by Charles S. Whitney, first appeared in print in 1940[2] in the Transaction of the American Society of Civil Engineers. Since this theory is a comparatively recent development, the literature on the subject is very brief. The Plastic Theory has been included, in addition to the Conventional Design Theory, in one standard text[4] on reinforced concrete design to date.

The Ultimate Theory of Reinforced Concrete Design was developed by Vernon P. Jensen, and was first published in 1943 in the Journal of the American Concrete Institute.[3] A pamphlet published by the Portland Cement Association[5] in 1951 gives the development of the Ultimate Theory and contains tables and diagrams designed to establish an easy transition from the conventional method to the new approach. Although the information

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[4]Peabody, Dean, Jr., Reinforced Concrete Structures, 2nd ed., New York, John Wiley and Sons, 1946.

[2]Whitney, Op. Cit. p. 5.

[3]Jensen, Op. Cit. p. 6.

[5]Portland Cement Association, "Ultimate Design of Reinforced Concrete", 1951, 18 pps.

developed in this pamphlet differs slightly in several points from Jensen's original hypotheses the presentation is excellent both from the theoretical and practical standpoint.

While technical literature in the field of hipped plate construction is abundant in Germany, very little information is available in the English language. Fortunately, an excellent presentation of the history and method of design was made in the Journal of the American Concrete Institute in 1947.[6] This article contains the basic approach and underlying theory of the hipped plate structure and contains an extensive foreign bibliography.

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[6]Winter, G. and M. Pei, "Hipped Plate Construction", Journal of American Concrete Institute, Vol. 18, 1947, p. 505.

CHAPTER II

PROCEDURE USED IN THE INVESTIGATION

Selection of Structure

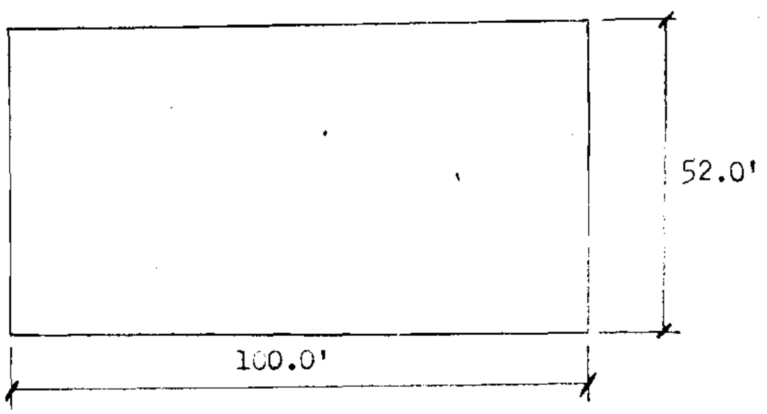
The base structure selected for the design was chosen on the basis of three general requirements:

(1) The proportions of the structure should be conducive to hipped plate construction;

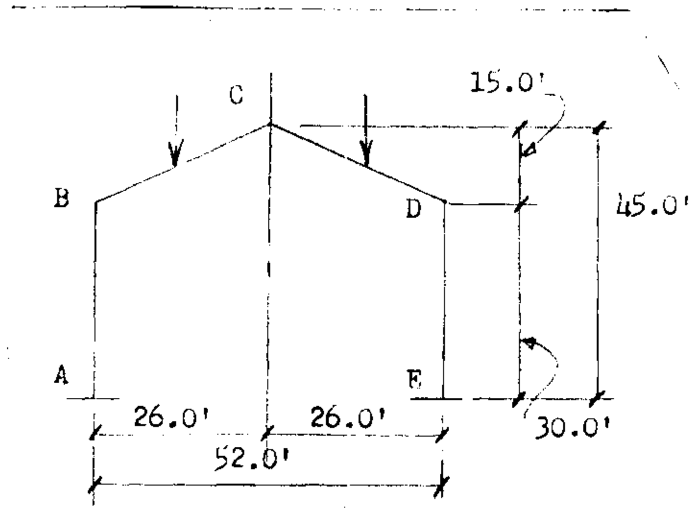
(2) The structure should be of a type that is practical and in current usage;

(3) The structure should be of sufficient size to show a definite comparison between the methods of design.

Based on the above requirements, the structure selected for the analysis was of the gable frame type with a side height of 30' 0" and a center height of 45' 0". The overall dimensions were 52' 0" x 100' 0". The general proportions are as shown in Figure 2.



(a)



Dimensions of Structure

Figure 2

The structure selected was of a type commonly used for warehouses, airport facilities and general industrial structures. For the rigid frame construction, two hinged type gable frames of variable section were used. The use of hinges and a variable section was selected for economic reasons because less material is required and the analysis is simpler than for a fixed-end structure. The roof construction was composed of concrete roof joists (or tee beams). The rigid frames were spaced 20' 0" apart along the longer dimension of the structure. The virtual work method was used in the analysis of the gable frames under the various conditions of loading which will be described later.

Method of Analysis - Rigid Frame Construction

A statically indeterminate structure is one in which the reactions or stresses produced by the conditions of loading cannot be determined from the conditions of static equilibrium. The unknown conditions can be in terms of any number of direct forces or couples, as for reactions and moments, or in any combination. The procedure of solution by the method of work is to replace the unknowns with statically equivalent forces which are numerically equal to the redundant, or unknown, quantities. The statical equivalent forces are, in general, denoted as "X". If the forces, "X", are then removed from the structure, a statically determinate structure will result. However, there will be a certain displacement of the structure at the points of application of the statically equivalent "X" forces due to the externally applied loadings. These displacements can be calculated from the fundamental formulas of statics. The principle of superposition, which states that the effect of a system of forces acting on a given structure will be equal to the sums of the effects produced by the individual forces, is next applied to the solution for the unknown "X" forces. If the displacement of the structure under externally applied loadings is denoted as $\underline{S}_{ao}$, the general equation for evaluating the unknown values would be:

$$\underline{S}_a = \underline{S}_{ao} + X_a\underline{S}_{aa} + X_b\underline{S}_{ab} + X_c\underline{S}_{ab} + \ldots \tag{4}$$

where: $\underline{S}_a$ = net horizontal movement of point A which, in the case of unyielding supports, is numerically equal to zero.

$\underline{S}_{ao}$ = $\frac{M}{EI}$ yds = horizontal movement of point A when the structure is unrestricted to horizontal movement under externally applied loadings.

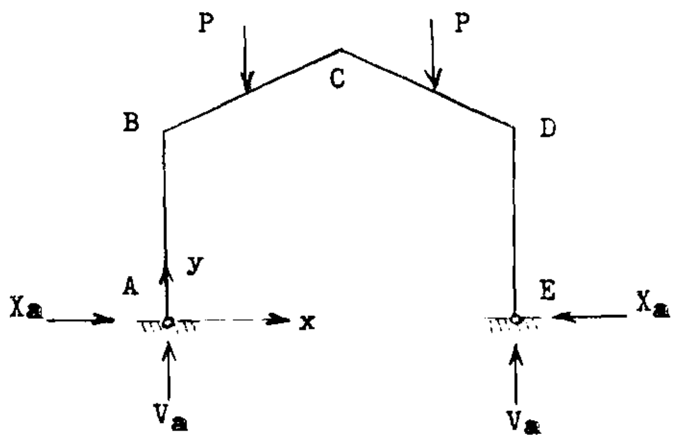
$\underline{S}_{a-a} = \quad \frac{y^2 ds}{EI}$ = horizontal movement of point A due to a unit force at A.

$\underline{S}_{a-b}$, $\underline{S}_{ac}$ = movement or rotation of point A due to unit force or unit couple at A.

X_a, X_b, X_c = numerical value of forces or couples at point A.

For the conditions of this investigation there is only one redundant, the horizontal force at the hinges (Fig. 3), and so the displacement equation will be

$$\underline{S}_a = \underline{S}_{ao} + X_a\underline{S}_{aa} = 0 \qquad (5)$$

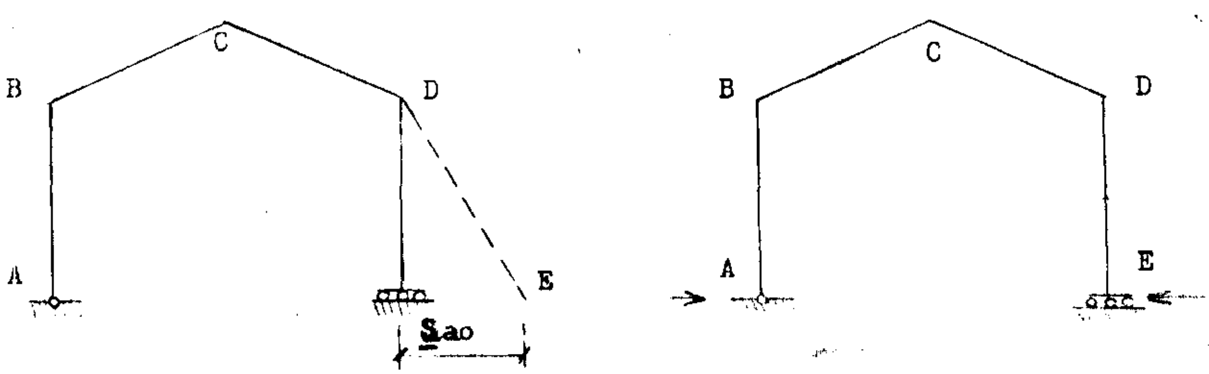


The Base Structure

Figure 3

The method is to think of the hinge at one end of the frame as being replaced with rollers, and to calculate the horizontal movement (Fig. 4) that would result from the external loadings, P, only. This displacement may be calculated from the relationship

$$S_{ao} = \frac{Mo}{EI} yds \tag{6}$$



Condition $X_a = 0$

Figure 4

The horizontal movement of the same point due to a force of X equal to unity (Fig. 5) will be

$$S_{a-a} = \frac{y^2 ds}{EI} \tag{7}$$

In order that the conditions of equilibrium be satisfied for a structure in which the supports do not yield there can be no horizontal movement at the hinges, and so (from Equation 5),

$$X_a = - \frac{Sao}{Saa} = \frac{\frac{Mo\ yds}{EI}}{\frac{y^2 ds}{EI}} \tag{8}$$

Equation 8 is the general expression for evaluating the redundant, X_a. After the redundant, X_a, has been evaluated the moment at any point, X, on the frame section produced by an external load, P, is

$$M_x = M_{ox} + M_{ax}'y \tag{9}$$

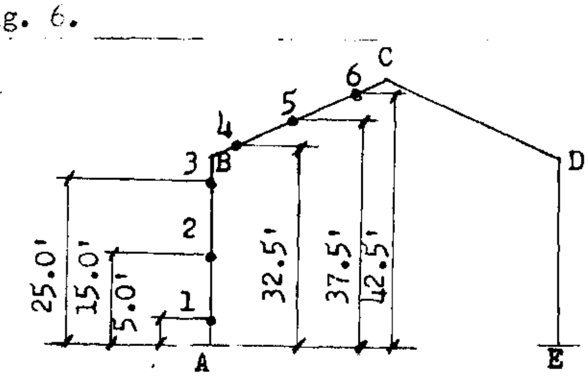
where M_x = combined bending moment at any point, X, on the frame.

M_{ox} = Bending moment at point X, due to the external loads, P, when the condition $X_a = 0$.

M_{ax} = Bending moment at point X due to the force X_a = unity.

y = Vertical distance from base axes to point X.

For a constant section frame of the same material throughout, the term EI may be neglected since it is constant for all points on the frame. However, for an arch of variable section it is necessary to divide the arch axis into a convenient number of ds-segments which may or may not be equal in length. The center of each such ds-length is then numbered for convenience. In this investigation the ds-lengths were selected as ten feet long. The base structure will then be as shown in Fig. 6.



Base Structure for Variable Section

Figure 6

Since the term ds/I appears in both the numerator and denominator of Equation 8 and since the frame is symmetrical about the vertical axis and contains ds-segments of equal length, the moment of inertia may be determined at the mid-point of the ds-segments rather than at the center of gravity.

The virtual work procedure as described above applies to only the given base structure. Several excellent texts containing more thorough discussions of the method and its applications are available for a more detailed description.[1,2]

Method of Determining Section Dimensions

The method used to determine the depth of arch at various points along its axis was to first analyze the base structure as a three hinged constant section arch with hinges at points A, C and E (Figure 6). The structure was then statically determinate. If convenient values (12", 16", 18", 24", etc.) are then chosen for the width and depth of arch section, both the horizontal and vertical reactions may be evaluated by statics. The moments at the centers of the ds-lengths are next determined. Then, taking the Conventional theory as an example, the depth of section at any point on the axis is

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[1]Parcel, J. and G. Maney, Statically Indeterminate Stresses, 2nd Ed., New York, John Wiley and Sons, 1950.

[2]Fife, W. M. and J. B. Wilbur, Theory of Statically Indeterminate Structures, 1st Ed., New York, McGraw-Hill Book Company, 1937.

$$d_x = \frac{M_x}{Kb} \tag{10}$$

where: d_x = section depth at any point, x.

M_x = bending moment at point x.

K = design constant for reinforced concrete (Equation 3).

This preliminary analysis can be extended to determine the width of section most suited to an economical spacing of reinforcing bars. The method is to first obtain the required steel areas at the points on the arch section using the formula

$$A_s = \frac{M_x}{f_s j d_x} \tag{11}$$

where: f_s = design strength of reinforcing bars.

j = section constant dependent on quality of materials (Equation 2).

A_s = area of reinforcement.

The number, size and spacing of the steel bars required can then be determined by referring to Tables 5 and 6 in the Reinforced Concrete Design Handbook[3] or by trial and error process by fitting various

[3]American Concrete Institute, "Reinforced Concrete Design Handbook", 1st ed., Detroit, published cooperatively by American Concrete Institute, Portland Cement Association, Concrete Reinforcing Steel Institute and Rail Steel Bar Association, 1948.

bar diameters plus horizontal clearance into the section width. The limiting width of section was taken as that which could best accomodate the steel requirements at the most critical sections with no more than two rows of steel bars. Even though the bending moment used in the preliminary calculations was not obtained by the consideration of all possible loading conditions, it does present a good indication of the section width (and depth) required for the maximum, or critical, loadings. In this investigation, the section width determined by the Conventional analysis was also used for the Plastic and Ultimate designs -- this was done to establish the depth of section required for a specified width for the three rigid frame designs.

Loading Conditions Assumed in the Design

The loading conditions assumed in the rigid frame analyses in this investigation are those used in current practice for this type of structure. They may be described as follows:

a. Dead load.

b. Live load on full span.[4]

c. Live load on left half span only.

d. Live load on right half span only.

e. Live load on center half span only.

f. Wind load acting toward the right against the left half of the frame.

g. Wind load acting toward the left against the right half of the frame.

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[4]The span referred to is the inclined portions of the frame, BCD, in Figure 3.

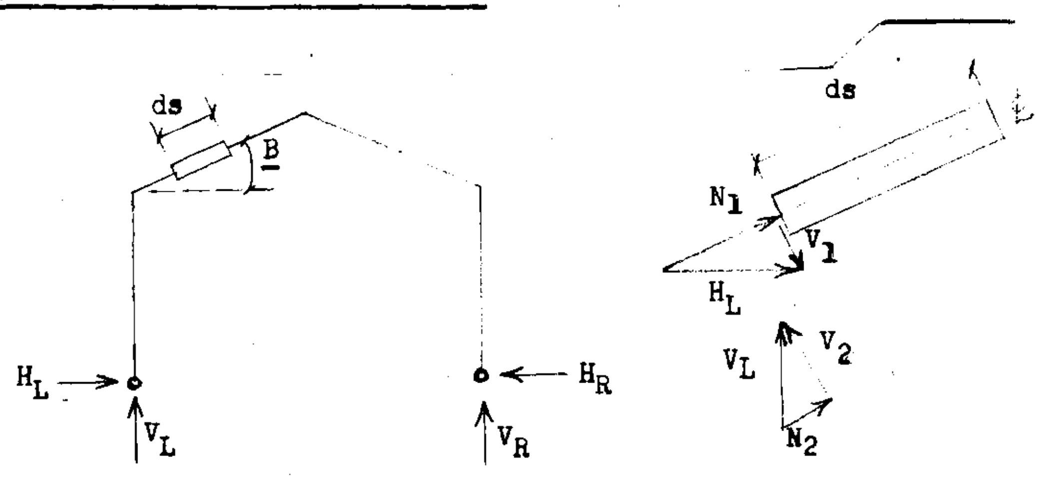
h. A temperature variation of $\pm$ 60° F.

As the structure is symmetrical about the vertical axis the number of analyses for each design theory can be reduced over those listed above. For example, the numerical value for the horizontal force, X_a, will be the same for both conditions of wind loading, while the bending moments produced with only the wind forces acting will be of the same magnitude for either side of the structure according to the point of application (that is, the moments produced in the frame ABCDE with the wind from the left, Fig. 3, will be numerically equal to the moments produced in EDCBA with the wind from the right). It is thus possible to obtain the combined effect of wind action on the structure for both directions of application with one analysis. The principles of symmetry and anti-symmetry may be applied to the results of the analysis for the live load full span to directly determine the "X_a" horizontal force for the live load on the left and right half of the span only. In the latter case the horizontal force, X_a, for the live load acting on the left or right half span only, will be one-half the value determined for the live load acting on the full span. Also, the combined moments resulting from the temperature variation of $\pm$ 60° F. need be determined for the rise or fall condition alone; the opposite movement will produce moments identical in magnitude but of opposite sign from those already obtained.

For the analyses of this investigation the dead load consisted of the weight of the concrete roof joists carried to the respective frames plus the weight of the frames themselves. The live load was considered to include loadings likely to be imposed during the construction, or service life, and was taken as 20 psf. Wind loads, like live loads, are inclined to be variable in nature, and the assumed value for this investigation was taken as 20 psf. A temperature differential of 60° F. was considered in these analyses. The coefficient of thermal expansion for both concrete and steel was assumed to be 0.000006 feet per 1° F.[5]

In the designs of this investigation the procedure was to first determine the bending moments produced at each section by each of the loading conditions described previously. The maximum moment that can act on any section is then the sum of the largest moments produced under each loading condition.

Determination of Thrust and Shear



Thrust and Shear

Figure 7

The method of obtaining the thrust and shear at the various midpoints of the ds-segments on the span may be described as follows:

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[5]Sutherland, H. and R. C. Reese, Reinforced Concrete Design, 2nd ed., New York, John Wiley and Sons, 1951, p—44.

1. Obtain the values V_L, V_R, H_L and H_R for each loading condition.

   The values H_L and H_R correspond to the redundant, X_a, described previously. The values V_L and V_R are the vertical components of the reactions produced by any loading combination.

2. For the inclined member, BC, (Figure 17), the thrust on any length, <u>ds</u>, is

$$H \cos \underline{B} + V \sin \underline{B} \qquad (12)$$

The component of the reactions acting normal to the member (the vertical shear) at any section is

$$V \cos \underline{B} - H \sin \underline{B} \qquad (12a)$$

For the vertical member, AB, the thrust will be simply V_L, while the vertical shear will be H_L (for most conditions of loading). For members CD and DE the method is identical.

It is convenient to arrange these values in tabular form (covering all loading conditions) and select the loading combination giving the maximum values for each point of investigation on the frame. The tabulations for the three design theories are contained in the Appendix.

<u>Calculation of Steel Areas</u>

After the maximum bending moment and thrust at each section are determined, the method of computing the steel requirements is essentially that for a section subjected to combined bending and axial load. A Standard text on rigid frame construction[6] contains a convenient

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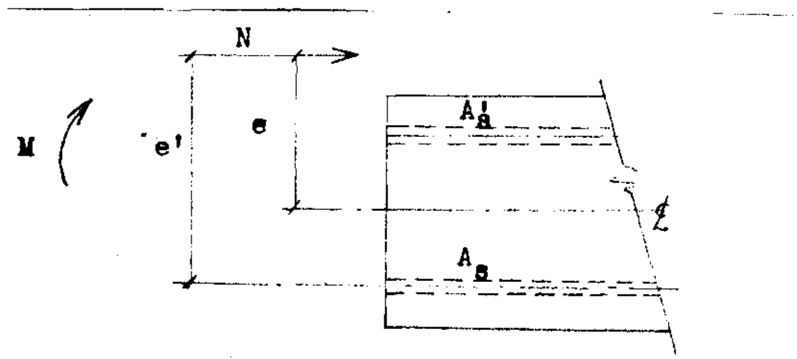
[6]Hayden, A. and M. Barron, <u>The Rigid Frame Bridge</u>, 3rd ed., New York, John Wiley and Sons, 1952.

tabular form for presenting the information required for determining steel areas, and a similar tabulation was employed in this investigation.



Forces Acting on Section

Figure 8

The forces acting on a typical arch section are shown in Figure 8. The moment and thrust acting at any section can be replaced by an equivalent eccentric load, N, acting at a distance

$$e = \frac{M}{N} \tag{13}$$

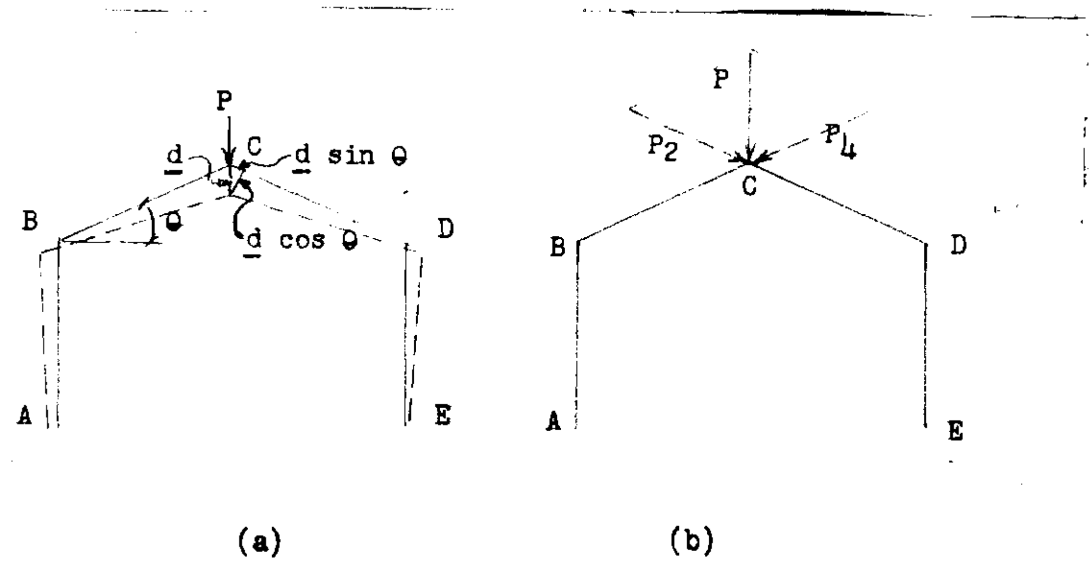
from the center-line of the section. The values "e" and "N" are used in determining the equivalent bending moment which will have the same effect on the section as the values "M" and "N" acting together. The method is developed in detail in all standard texts on reinforced concrete design. The "Reinforced Concrete Design Handbook"[3] contains a thorough discussion of the method plus tables to facilitate its application with the Conventional design theory. This method is

[3] American Concrete Institute, Op. Cit., page 17.

applicable to each of the three design theories discussed in this investigation and is described in detail in the Appendix.

Method of Analysis - Hipped Plate Construction

Hipped plate construction is a type of box-like construction using continuous slabs with end diaphragms (which act as the front and back ends of the box). No beams or girders are required despite the considerable spans possible with this type of construction.



Deflections and Load Resolutions

Figure 9

Figure 9(a) shows a hipped plate structure of the type selected for this investigation with the load, P, acting. Due to the action of the diaphragm connecting AB and DE, the deflection, d, produced by this load can occur only in the manner shown. The total deflection, d, resolves itself into two components, one being perpendicular to the inclined slab (d cos Θ), the other being parallel to that slab (d sin Θ).

While the inclined slabs are extremely flexible in bending due to loads acting perpendicular to the axes BC and CD, they are very rigid and act as beams between end diaphragms with respect to loadings parallel to BC and CD. It can be seen that the deflection (d sin Θ) at the adjoining edge, C, will be of the magnitude of inclined plates acting as beams between end diaphragms. Figure 9(a) greatly exaggerates the slight spreading that may occur with this construction. In a correctly designed structure this spreading is extremely small as it is resisted by the rigidity of the plates with respect to deflections in their own planes, plus the action of the tie beams.

Figure 9(b) illustrates the manner in which the load, P, is resisted by the structure. As any slab spanning a large distance is very flexible in a direction perpendicular to its span but very rigid in its beam action, a load, P, will resolve itself into two components parallel respectively to the two adjoining slabs. In the symmetrical case shown, the forces P_2 and P_4 will be carried in flexure by the inclined slabs.

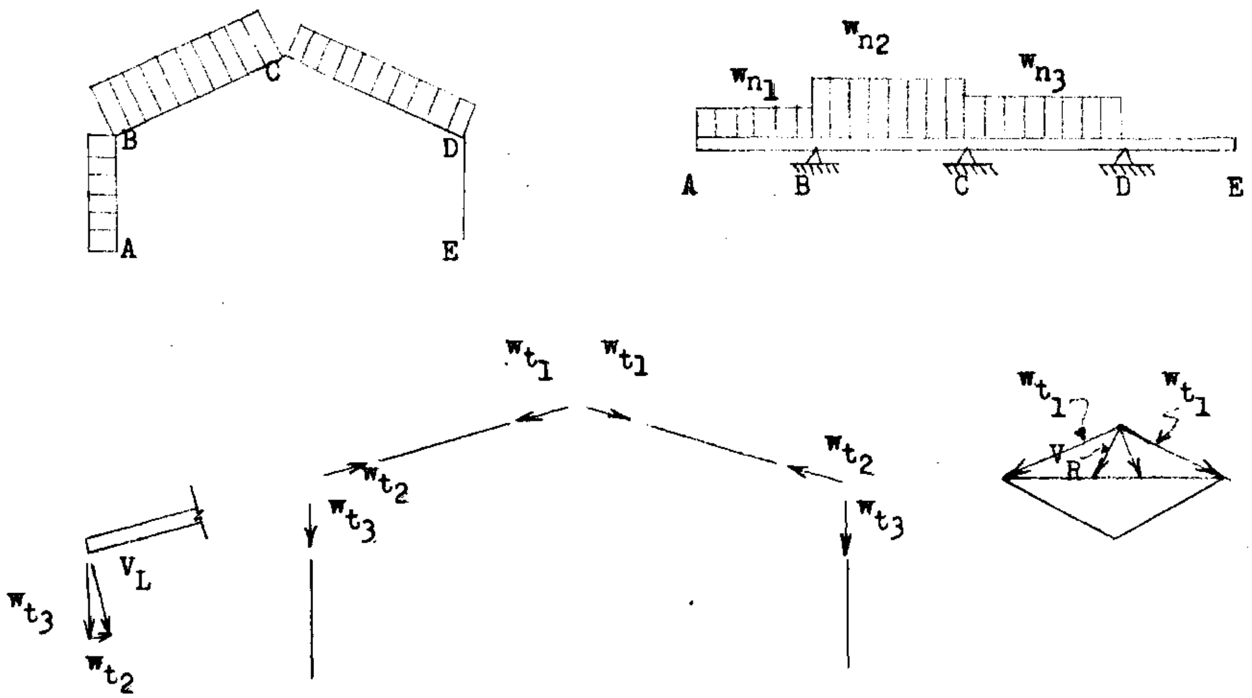
Figure 10(a) illustrates a structure under loading. The method of analyzing a hipped plate structure of this type may be divided into three parts:

(1) Analysis of slab action for loads perpendicular to the spans AB, BC, CD and DE.

(2) Analysis of the plate action in which the "w_t" loadings (Figure 10c) are taken by the slabs in beam action.

(3) Analysis of the shear and normal stresses in the plates

to determine the magnitude and direction of the principal tensile shearing stresses.



Load Resolution

Figure 10

For the first portion of the analysis the loadings acting on the structure are resolved into their components perpendicular to the axes AB, BC, CD, and DE. These loadings normal to the plane of the slab are denoted as "w_n" forces (Figure 10b). Since the depth of slab is unknown and must be determined by the trial and error process, the resolved normal loadings are placed on the structure in combinations reflecting the probable maximum moments in the slabs. For example, a probable combination would be as follows:

A — B	B — C	C — D	D — E
W. L.	D. L.	D. L.	0
	L. L.	L. L.	
	W. L.		

Similar combinations involving consideration of the live loading on only spans BC or CD, or the elimination of wind loadings may be devised. The slab is then analyzed to determine the maximum moments acting and the required depth of slab. The method of moment distribution was used in this investigation. Knowing the maximum moments acting in the slab it is then dimensioned and reinforced as a one-way slab in the conventional manner. From the previous analysis, the static equation

$$M = 0 \qquad (14)$$

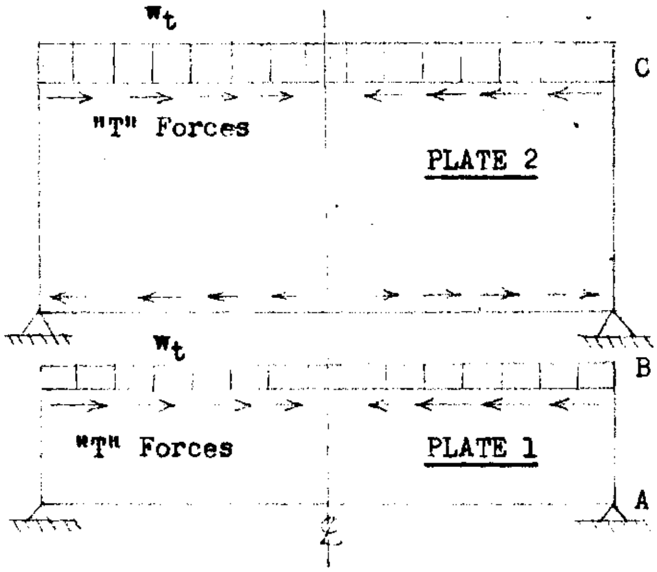
may be employed to determine the vertical reactions at B and C. While the effect of these reactions is conventionally indicated in a direction opposite to that in which the applied loads are acting, they are (for hipped plate construction) considered as acting at the junctions of the plates in the same directions as the applied loadings. For example, in a simple beam with uniform loading the vertical reactions are considered as acting upwards; here, the supports actually take the reactions as loadings on a beam and the direction of action is reversed from the conventional nomenclature for vertical shear reactions. Figure 10(c) illustrates the manner in which the vertical reactions, V_L and V_R, are transmitted to the plates in plate action. For convenience, the plate action in BC (Figure 10a) will be denoted as Plate 2, and that in AB as Plate 1. Plate 1 will then be acted on by the "w_{t3}" loads (Figure 10c) plus the component of the plates weight in the plane of the "w_{t3}" loadings. Similarly, Plate 2 will be acted upon by the loadings, w_{t1} and w_{t2}, plus the component of its weight in the plane of these loadings. The algebraic sum of the loadings on each plate will produce the total load, w_T (Figure 11), acting. The bending moment will be a maximum at the span midpoint and will be equal to

$$M_e = w_T L^2/8 \tag{15}$$

This value is termed the "central bending moment of each plate. Since the strains (and therefore the stresses) must be equal at the adjoining edges, or junctions, of any two plates, horizontal shearing forced will exist at these points. The shearing stresses produced by these forces at the junctions are equal to the vertical shearing stresses produced by the uniform loadings. They will vary (for this type of loading) in the same manner as in a beam, in a triangular manner being a maximum at the supports and of zero intensity at mid-span. In this discussion the shearing forces along the edges (pounds per foot) will be denoted as "T" forces, the shearing stresses along the edges as " " stresses (T/12b). If a section were cut through the plates at mid-span (Figure 11) as a free body diagram, the summation of the "T" forces and the bending moment acting would produce equivalent opposing forces to maintain the conditions of equilibrium (Figure 12). The forces existing at the midpoints of the plates are shown in their true direction of action in Figure 12. The longitudinal edge force, N, is equal to

$$N = \quad T\,dx \;=\; \quad T\,dx \tag{16}$$

and will vary parabolically along the span. The force, N, may be



Forces Acting in Plates

Figure 11

determined from the fact that the strains at the adjoining edges of the two plates are equal; the process is somewhat simplified by the use of a shear distribution analogy similar to the moment distribution method.[7] The shear distribution analogy was used in this investigation. The analogy, briefly described, is dependent on the ratio, M_o/h, which produces a term analogous to the fixed end moments of the moment distribution method for each plate. This term will be referred to as the "fixed end shear". Determining the "fixed end shear" for each plate and distributing these values in the conventional moment distribution procedure

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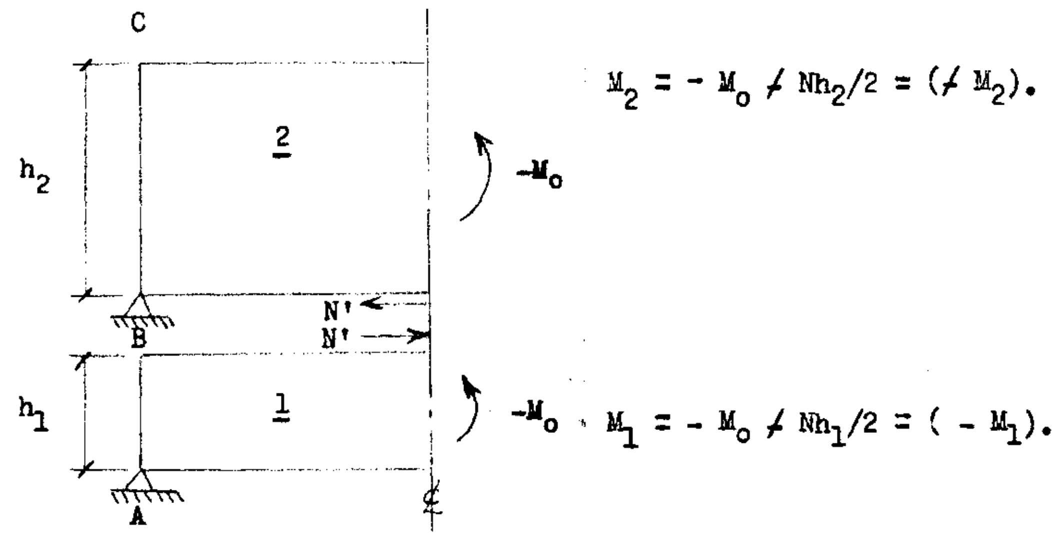
[7]Winter, G., and M. Pei, "Hipped Plate Construction", Journal of American Concrete Institute, Vol. 18, 1947, p - 505.

will result in the value of "N" at the plate edges. The resulting



Forces at Plate Midpoint

Figure 12

moment (see Figure 12) at the midpoint of each plate will then be equal to

$$M = M_o \pm Nh/2 \tag{17}$$

Knowing the direction and magnitude of "M" and "N" for the individual plates, they are designed in the conventional manner for reinforced concrete members subjected to combined bending and axial loading. Both "M_o" and "N" will vary parabolically along the span length, being a maximum at mid-span, and can therefore be determined at any point along the span. The edge forced, T, can be obtained from these values by equating the area of the shear diagram due to the edge forces to the longitudinal force, N, where

$$T_{end} \times \frac{L}{4} = N \tag{18}$$

and

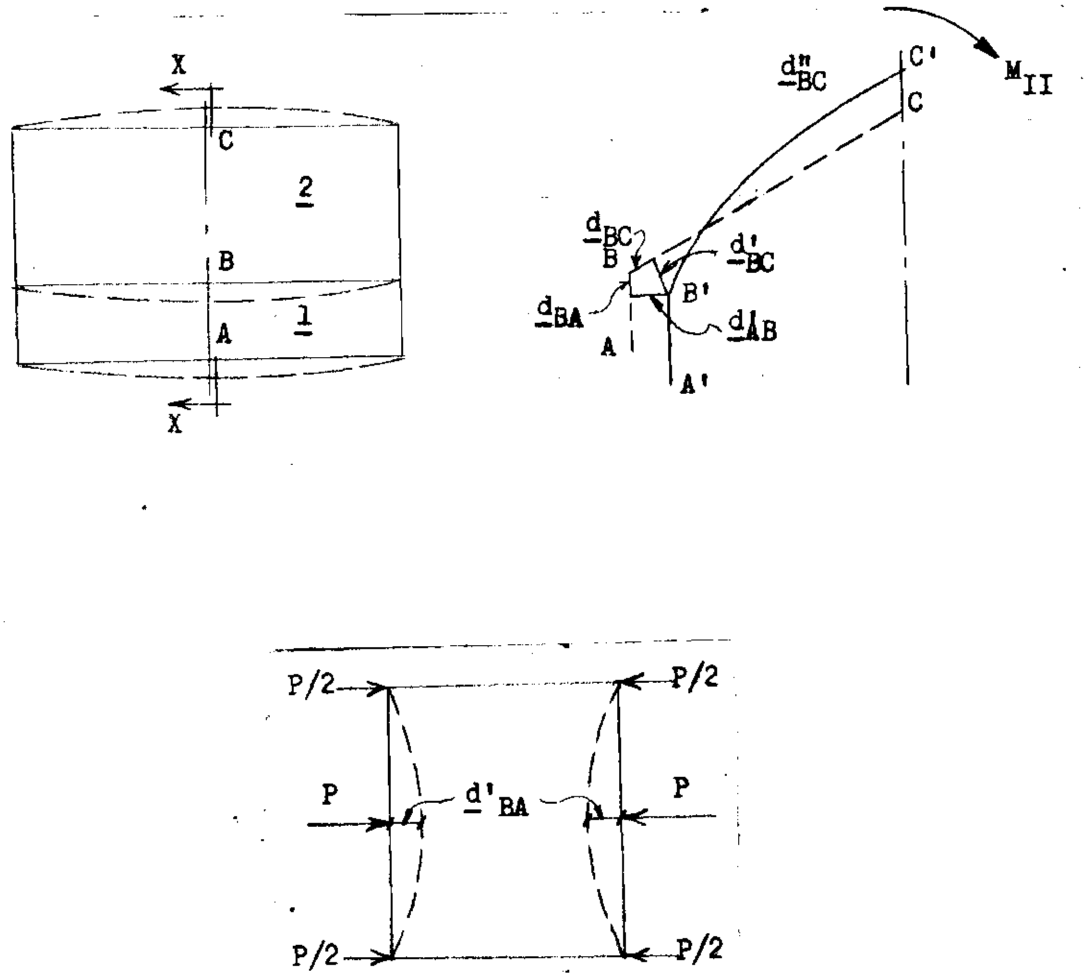
$$T_{end} = \frac{4N}{L} \tag{18a}$$

From the value, T_{end}, the edge forces, T, along the span can be determined (Figure 41, Appendix). Also, the shearing stress, τ, may be obtained from the relationship

$$\tau = \frac{T}{12b} \tag{19}$$

The shearing stress is used in the third stage of the analysis.

The moments resulting from the combination of the "M_o" and "N" values in the plates will produce deflections in the plates. The deflections will be a maximum at the midpoints of the plates, as for a simple beam with uniform loading, but their direction will depend on the direction of the previously determined bending moments. Figure 13 illustrates the deflection diagram at the plates midspan for the resulting bending moments shown in Figure 12. Point C can move only up and down at the midpoint of Plate 2 — the resulting bending moment in Plate 2 (Figure 12) shows that it will move upwards to C'. Similarly, points B and A will move downward and inward to B' and A' at the plate midpoints. The supporting edges of the plates (Figure 11) are assumed to be unyielding because of the resistance offered by the diaphragm at each end. However, the distortion shown in Figure 13 will produce a bending moment at point C'. The effect will be that of a beam fixed



Deflections at Mid-span of Plates

Figure 13

at one end (C') and guided at the other due to the restraint offered by the diaphragm. The deflections illustrated in Section X-X (Figure 13) can be obtained directly from the magnitude of the deformations "$\underline{d}_{BC}$" and "$\underline{d}_{BA}$" at the plate midpoints. Knowing the total deformation in Plate 2 ($\underline{d}''_{BC} \neq \underline{d}'_{BC}$) the bending moment at C' may be determined where

$$M_{II} = \frac{3EI\underline{d}}{L^2} \qquad (20)$$

The term M_{II} is denoted as the "secondary bending moment" in the plates. The reactions produced by M_{II} at points B and C are $\pm M_{II} / h_2$. They, in turn, are resolved into their respective plate loadings, w_t, by a method similar to that illustrated in Figure 10. The resulting plate loadings will vary parabolically along the plate edges (corresponding to the plate deflection under the external loadings described previously). The central bending moments, M_o, are obtained from the parabolic loading condition and the resulting longitudinal edge force, N_2, is determined through the application of the distribution analogy given before. Should the force, N_2, produced by the effect of the plate deflections increase the previous value determined, N, the value $N + N_2$ is considered as the thrust in the design of the plates. A similar procedure applies to the resulting bending moments.

The force in the end diaphragm or tie beam will be a compressive force for a hipped plate structure having the deflection characteristics of Section X-X, Figure 13. The deflection, $\underline{d}'_{BA}$, will be the same as that produced by some unknown concentrated load, P, at the midpoint of Plate 1. This unknown loading will produce reaction in the tie beams equal to one-half "P" for each half of the structure; the total force in each tie beam will then be "P" (see Figure 13b). The force, P, at the span midpoint can then be resolved into plate loadings in terms of "P" (Figure 45b, Appendix) and the resulting central bending moment, M_o, and longitudinal force, N_3, can be determined. By equating the deflections induced by the load, P, to those previously determined in the secondary bending moment procedure the magnitude of the force "P" is found. The tie beams are then designed as members subjected to the bending moment produced by their dead weight plus the thrust, P.

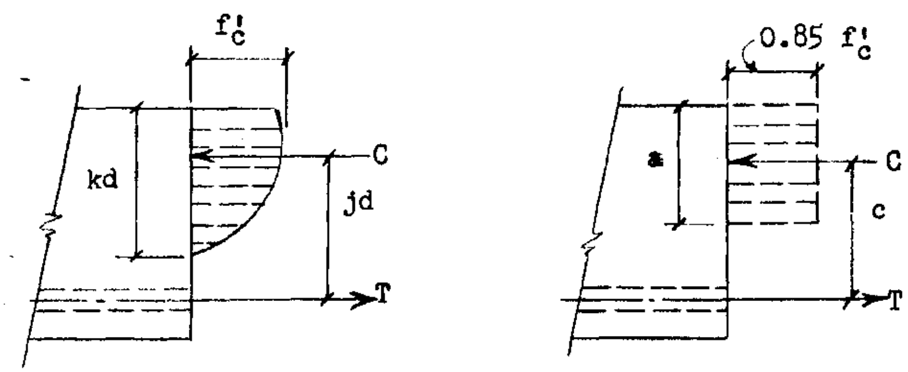
The third stage of the analysis involves the determination of the tensile stresses acting in the plates. An extensive discussion and illustration of the procedure involved is contained in the Appendix and the method will not be presented at this point.

Hipped plate structures may consist of solid slabs with openings for windows and doors or they may be erected on columns with non-load bearing walls underneath. In the latter case, which was used in this analysis, the columns and tie beam form a bent. The procedure was to first design the hipped plate structure completely and to then analyze the end diaphragm plus columns as a bent. The analysis of the bent will involve the maximum loading of the hipped plate structure plus the wind forces on the structure. A discussion and illustration of the procedure is made in the section of the Appendix covering the hipped plate design.

APPLICATION OF DESIGN THEORIES

The Plastic Theory of Reinforced Concrete Design

In his original article on the Plastic Theory of reinforced concrete design[8] Mr. Whitney stated that the theory of elasticity as applied to reinforced concrete, is "too inflexible and inaccurate to be entirely satisfactory". He proposed the adoption of the Plastic Theory which considers the true characteristics of the material as determined by research. The design equations developed, completely eliminate the modular ratio "n", are much simpler to apply than the standard formulas, and give a better agreement with test results. They are based on the cylinder strength of concrete, f'_c, and the yield strength of steel, f_s.



True and Equivalent Compression Distribution

Figure 14

[8]Whitney, Charles S., "Plastic Theory of Reinforced Concrete Design", Transactions, American Society of Civil Engineers, Vol. 56, 1940, p-1749.

Figure 14(a) represents the compression distribution on a rectangular beam section at failure as determined from the shape of a cylinder stress-strain curve. The total compression, C, is the area bounded by the curve and its line of action is through the center of gravity of the area. If the actual area is replaced by an equivalent rectangular area of width equal to $0.85f'_c$ and depth equal to "a" as shown in Figure 14(b), the location of the center of gravity of this rectangle corresponds closely with that of the actual area. The rectangular block is not substituted for the true distribution but presents a simple means of approximating the effect of the actual distribution.

Figure 14(b) illustrates the basic representation and nomenclature of the Plastic Theory. The term, a, does not correspond to the depth to neutral axis, kd, used in the conventional theory—it is, rather, a distance determined from the strength of the materials.

If the beam contains an insufficient amount of steel such that primary failure will occur in the tension steel, the beam is said to be under-reinforced and the resulting value of "a" will be

$$a = \frac{A_s f_s}{0.85\ bf'_c} = \frac{A_s m}{b} \qquad (21)$$

or, it can be written

$$\frac{a}{d} = pm \qquad (21a)$$

where, A_s = area of tensile steel

f_s = yield point stress in steel

b = width of beam

f'_c = standard concrete cylinder strength

p = A_s/bd, or steel percentage

m = $f_s/0.85\ f'_c$

The lever arm of the steel reinforcement may be written

$$c = d - \frac{a}{2} = d - \frac{A_s m}{2b}$$

or,

$$\frac{c}{d} = 1 - \frac{pm}{2} \tag{22}$$

The ultimate resisting moment as controlled by steel failure can be written

$$M = cA_s f_s = A_s f_s (d - \frac{A_s m}{2b})$$

or,

$$\frac{M}{bd^2} = pf_s\ (1 - \frac{pm}{2}) \tag{23}$$

For an under-reinforced beam

$$M = 0.85\ f'_c ab(d - \frac{a}{2}) \tag{24}$$

from which

$$\frac{a}{d} = 1 - 1 - \frac{2.35M}{f_c'bd^2} \tag{25}$$

and

$$\frac{c}{d} = \frac{1}{2}(1 + 1 - \frac{2.35M}{f_c'bd^2} \tag{26}$$

The required steel area is

$$A_s = \frac{M}{cf_s} \tag{27}$$

From the results of beams tested to failure Mr. Whitney concluded that, for balanced design,

$$\frac{a}{d} = 0.537 \tag{28}$$

and

$$\frac{c}{d} = 0.732 \tag{28a}$$

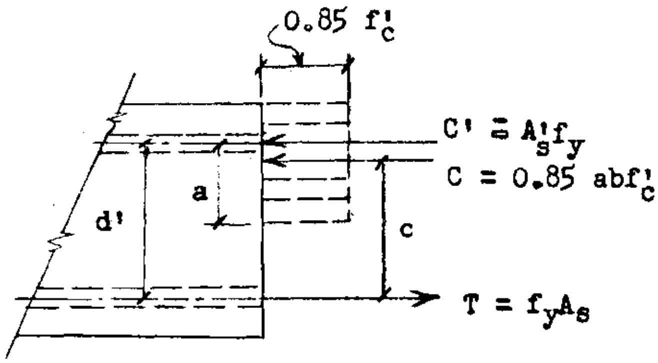
For the condition of balanced design

$$\frac{M}{bd^2} = 0.85\ f_c'\ \frac{a}{d}\ (1 - \frac{a}{2d}) = \frac{f_c'}{3} \tag{29}$$

From Equation 21(a), the critical percentage of steel required to develop the full compressive strength of the concrete is

$$p_o = 0.456\frac{f_c'}{f_s} \tag{30}$$

The procedure for balanced design is to determine the section dimensions from Equation 29, and apply the critical percentage of steel (Equation 30) immediately. All of the foregoing equations are independent of the modular ratio, n, and are much simpler in application than the conventional design formulas.



Compression Reinforcement

Figure 15

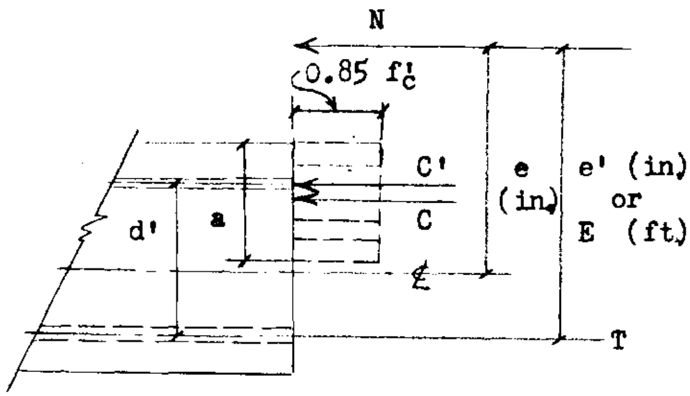
Figure 15 illustrates the representation and nomenclature for a section reinforced for compression. The ultimate bending moment in compression is computed by adding the moment of the steel compressive stress to that of the concrete stress. Then, if the section is fully reinforced in tension, the ultimate bending moment at which the beam will fail in compression will be

$$M = \frac{f_c' b d^2}{3} + d' A_s' f_s \tag{31}$$

from which

$$A_s' = \frac{M - f_c' bd^2/3}{f_s d'} \qquad (32)$$

The procedure in this case is to determine the area of compression steel required for the given bending moment from Equation 32. For a section fully reinforced in tension, the tensile steel area is computed from Equation 27. In his original article, Mr. Whitney also develops the equations to be applied to beams under-reinforced in tension. Since this case is not frequently encountered in the usual design procedure they will not be given here-in.



Bending and Direct Load

Figure 16

Figure 16 illustrates the representation and nomenclature for a section subject to bending and direct load. Since the strength of the compression side of the section will be the same as that when subjected to bending alone, Equation 31 may be used to predict the resistance to compression failure. The ultimate compressive moment is

$$M = P(e + d - \frac{t}{2}) = \frac{bd^2f'_c}{3} + d'A'_sf_s \qquad (33)$$

from which

$$N = \frac{2A'_sf_s}{1 + 2e/d'} + \frac{btf'_c}{3te/d^2 + \frac{6dt - 3t^2}{2d^2}} \qquad (34)$$

Equation 34 gives the theoretical maximum value of the direct load when the eccentricity of the load is greater than that of the compressive resisting forces. For smaller eccentricities it is necessary to substitute 1.178 for the second term in the denominator of the second portion of the expression.

In this investigation the writer used the principle of an equivalent eccentric loading where applicable to determine the steel areas required for sections subject to bending and direct load. Assuming that a section requires compression steel, that the equivalent eccentric load lies outside of the section, and that the section will be fully reinforced in tension

$$Ne' = f_sA'_sd' + \frac{bd^2f'_c}{3} \qquad (35)$$

The term, $\frac{bd^2f'_c}{3}$, denotes the ultimate compressive moment the section can carry if balanced design is assumed, and may be written "$M_{ult.}$". Then,

$$f_sA'_sd' = Ne' - M_{ult.} \qquad (36a)$$

and since

$$Cc_o = M_{ult.}$$

$$C = \frac{M_{ult.}}{c_o} \tag{36b}$$

where c_o is equal to 0.732d for balanced design. Therefore,

$$Ne' = c_1\left(\frac{Ne' - M_{ult.}}{d'} + \frac{M_{ult.}}{c_o}\right) \tag{36c}$$

and

$$c_1 = \frac{Ne'}{\frac{Ne' - M_{ult.}}{d'} + \frac{M_{ult.}}{c_o}} \tag{37}$$

where c_1 is the distance from the point of action of both compressive forces to the center of gravity of the tensile steel.

If the term, Ne', is less than or equal to $M_{ult.}$,

$$A_s' = 0$$

and

$$A_s = \frac{N(e' - c_o)}{f_s c_o} \tag{38}$$

If the term, Ne', is greater than $M_{ult.}$,

$$A_s' = \frac{Ne' - M_{ult.}}{f_s d'} \tag{39}$$

and

$$A_s = \frac{N(e' - c_1)}{f_s c_1} \tag{40}$$

In his article, Mr. Whitney also develops equations governing the design of square and round columns. As the presentation of the Plastic Theory here-in is sufficient to cover the procedure followed in this investigation, no additional formulas will be given.

The design approach used in the development of the Plastic Theory tends to give a smaller section with more steel than the Elastic Theory. This is advantageous because the weight and size of the completed structure is, today (1953), a major factor influencing the choice between steel or reinforced concrete. The foregoing equations also include the effect of shrinkage and some plastic flow strains in their development.

The Ultimate Theory of Reinforced Concrete Design[9]

In his original article on the Ultimate Theory[10] Mr. Jensen advanced the hypothesis that concrete possesses both an elastic and a plastic range. The elastic range was said to be characterized by the currently used modular ratio, n, while the plastic range was characterized by a new term called the plasticity ratio.

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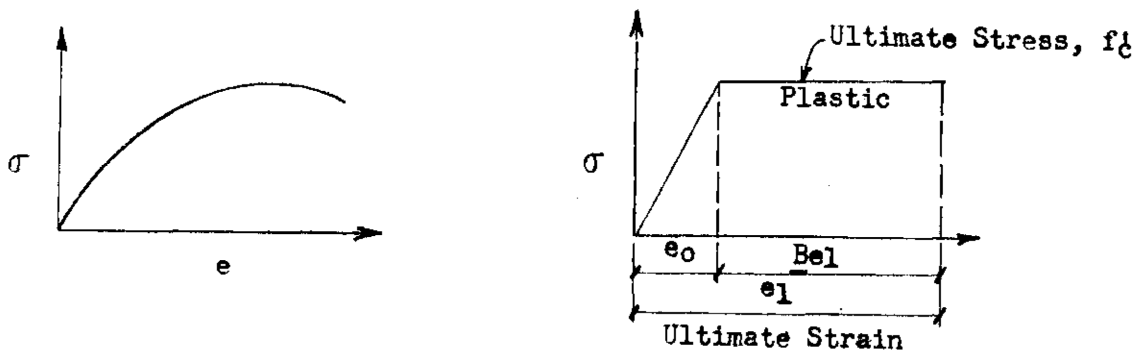
[9] The discussion as it pertains to this investigation will be limited to the procedure suggested in Ultimate Design of Reinforced Concrete, Portland Cement Association, 1951, 18 pps.

[10] Jensen, Vernon P., "Ultimate Design of Reinforced Concrete", Journal of American Concrete Institute, Vol. 14, 1943, p - 565.



Concrete Stress-Strain Diagrams

Figure 17

Figure 17(a) illustrates a typical stress-strain curve for a standard concrete cylinder in compression. The Ultimate Theory is based on an idealized stress-strain curve as shown in Figure 17(b).[11] The slope of the idealized curve in the elastic range is the initial modulus of plain concrete, E_c. Mr. Jensen states that this may be evaluated from the empirical formula for the modular ratio as

$$n = \frac{E_s}{E_c} = 5 + \frac{10{,}000}{f'_c} \qquad (41)$$

where f'_c = ultimate compressive stress of standard test cylinders

$E_s = 30(10)^6$ psi.

As the slope of the curve in the elastic range is f'_c/e_o, the strain

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[11]See also, Van den Broek, J. A., Theory of Limit Design, 1st ed., New York, John Wiley and Sons, 1945.

at the upper limit of the elastic range, e_o, can be determined.

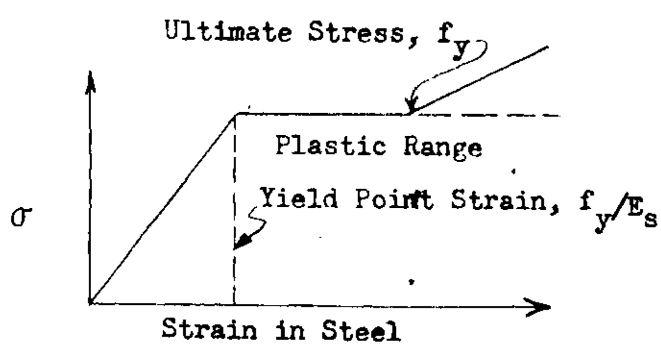
If the ultimate strain in the concrete is denoted as e_1, the plastic strain (Figure 17b) may be represented as $\underline{B}e_1$, where $\underline{B}$ is called the plasticity ratio. Mr. Jensen proposes that, for concrete made with gravel or crushed stone aggregate, the value $\underline{B}$ be taken as follows:

$$\underline{B} = \frac{1}{1 + (f'_c/4{,}000)^2} \tag{42}$$

From a consideration of Equations 41 and 42, with Figure 17(b),

$$e_1 = \frac{f'_c}{(1 - \underline{B})E_c} = \frac{5f'_c + 10{,}000}{(1 - \underline{B})\ 30(10)^6} \tag{43}$$

For concrete having a compressive cylinder strength of 4,000 psi. the values, e_o, $\underline{B}$ and e_1 will be 0.00100", 0.500, and 0.00200" respectively.

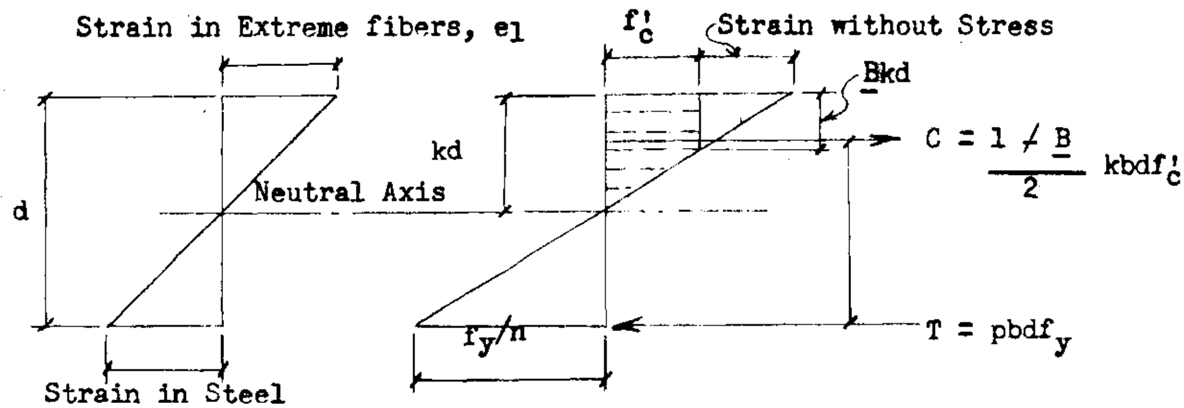


Idealized Steel Stress-Strain Diagram

Figure 18

The idealized stress-strain diagram shown in Figure 18 was suggested by Mr. Jensen for reinforcing steels having a marked yield point at some stress denoted as f_y. The plastic range is characterized by straight line and an inclined line. However, the characteristics of the curve near the yield point are affected by the rate of cooling during rolling, the composition of the steel, the size of bar and the speed of testing. The discussion contained in the Portland Cement Association publication (see footnote #9) suggests that it is justifiable, in view of the unknowns mentioned previously, to deviate from the stress-strain diagram shown in Figure 18, and substitute a straight line throughout the plastic range.

Although Mr. Jensen's theory states that failure occurs when the ultimate strain, e_1, exists in the extreme fiber, the Portland Cement Association prefers to define failure as the point at which the tensile steel stress first reaches the yield point. The latter definition is preferred because excessive cracking will not occur.



Stresses and Strains on Section

Figure 19

Figure 19 illustrates the representation and nomenclature for a section subjected to bending only. The customary assumption that plane sections remain plane after bending is upheld as shown. However, the stress distribution does not remain linear for plastic strains. As cna be seen from Figure 20, the conventional nomenclature for the various section properties is maintained in the Ultimate Theory.

When the condition of ultimate strain exists on the section the ultimate concrete stress will exist over a portion of the section equal to $\underline{B}kd$ (for Jensen's definition of failure). This can be shown from Figure 20.

It can be seen that the plastic strains on the section increase the amount of concrete in compression over that assumed in the conventional theory. The depth of section over which the concrete is equal to f_c' can be shown from Figure 20. Denoting this depth of section as "y", from similar triangles

$$\frac{\dfrac{Bf_c'}{1 - \underline{B}}}{y} = \frac{f_c'}{kd - x}$$

and therefore

$$y = \underline{B}kd \tag{44}$$

By similar manipulation the values of the section constants shown in Figure 20 are seen to exist. The ultimate resisting moment is

$$M = Tjd = (pbf_y)jd = (pf_yj)bd^2 \tag{45}$$

and the quantity

$$K = \frac{M}{bd^2} = pf_yj \tag{46}$$

as in the conventional theory. However, from the condition that T = C, where "C" is the area of the shaded portion of Figure 20, or

$$C = \frac{1 + B}{2} \, kbdf'_c \tag{47}$$

it can be shown that

$$k = 2pf_y/(1 + B)f'_c \tag{48}$$

Also, the lever arm, jd, the distance between "C" and "T" of Figure 20 is found from the relationship

$$j = 1 - \frac{2(1 + B + B^2)}{3(1 + B)^2} \tag{49}$$

The values of k, j and K shown by Equations 47, 48 and 49 apply only when the ultimate strain, e_1, is acting on the section. That is, when the yield point stress, f_y, has been developed in the tensile steel. When the tensile steel strain just equals f_y/E_s, special values denoted as k_o, j_o and K_o apply and may be obtained by substituting the value, p_o, into the previous equations, where

$$p_o = \frac{f'_c}{2f_y} \times \frac{1 + B}{1 + (B - 1)f_y/nf'_c} \tag{50}$$

The value, p_o, is the steel percentage for which the steel stress reaches f_y at the same instant the concrete strain reaches the magnitude e_1. It corresponds to the "balanced design" procedure of the conventional design theory. When the actual percentage of steel is greater than p_o, the steel is elastic all the way up to failure. Mr. Jensen states that, when this is the case, failure by compression is likely to occur suddenly without any warning. For that reason, it is not advisable to use steel percentages above the value of p_o.

If failure is defined as the point where the steel strain equals f_y/E_s, the area of concrete in compression may be reduced over that described previously. It may be said that, for this definition of failure, the depth of concrete stressed to the ultimate compressive stress will be some factor, x, times the depth to the neutral axis, kd, rather than the plasticity ratio, B, used previously. If, for convenience, the ratio f_y/f'_c is denoted as "r", Equation 48 may be written for this case, as

$$k = \frac{2pr}{1 + x} \tag{51}$$

It can be seen from Figure 20 that, for this definition of failure the value, x, is a variable dependent on the percentage of steel in a section. When the steel percentage is equal to p_o (Equation 51), the variable, x, will have its maximum value. It is at this point that the "balanced design" conditions for the Ultimate Theory will apply. Should the steel percentage be greater than p_o it is possible that the Ultimate Theory will no longer apply and then, the conventional, or elastic theory, can be used. The limiting percentage of steel, above which the Ultimate Theory no longer applies, is

$$p = \frac{n}{2nr + 2r^2} \tag{52}$$

When the percentage of steel is less than that obtained by Equation 52 the following section constants apply:

$$k = \frac{2pr^2 + n}{2r + n} \tag{53}$$

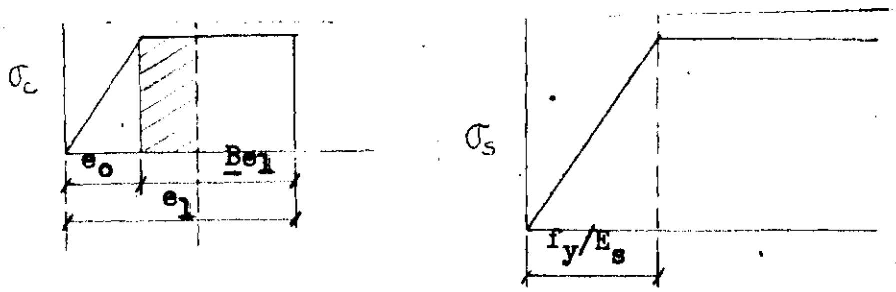
$$x = \frac{2pr - k}{k} \tag{54}$$

$$j = 1 - \frac{k}{3} - \frac{(2pr - k)^2}{6pr} \tag{55}$$

$$K = pf_y j \tag{56}$$

The Portland Cement Association publication contains tabular data for determining the section constants when the Ultimate Theory is used for design. With the use of these constants a section may be designed using the formulas now in use with the Elastic Theory.

The Ultimate Theory is a definite step forward in the effort to introduce the aspects of ultimate design to the engineering profession. It presents an excellent method for considering the ultimate properties of the materials used and agrees very well with existing test data.



Elastic and Plastic Effects on Stress Distribution

Figure 20

CHAPTER III

DISCUSSION OF RESULTS

A discussion of the results obtained from this investigation can best be described in terms of the relative amounts of steel and concrete required by each method of analysis. The presentation of the results will be divided into two phases, one phase covering rigid frame construction only and the second covering rigid frame with respect to hipped plate construction.

Rigid Frame Analysis

Before making a comparison of the conventional Plastic and Ultimate design theories a summary of the conditions under which the designs were made will be given. These conditions may be enumerated as follows:

1. Each design was made assuming six variable section gable type rigid frames; the frames were of the two-hinged type.
2. The conditions of live and wind loading were identical for all designs.
3. The exterior bents were made 9 inches wide and the interior bents 18 inches wide in the three designs.
4. Roof joists of identical section and length were used for all designs.
5. A factor of safety of 2.5 was assumed in the analyses using

the Plastic and Ultimate Theories; this gave allowable stresses of 20,000 psi. for steel and 1,200 psi. for concrete. Allowable stresses for the design by the conventional theory were taken as 20,000 psi. for steel and 1,350 psi. for concrete.

6. No increase in allowable stresses was made when the effect of wind loading was included in the determination of section dimensions.
7. The scope of this investigation covers the super-structure only.
8. Identical section depths were used in the designs using the Plastic and Ultimate Theories.
9. The rigid frame sections in all designs were reinforced with #11 steel reinforcing rods.

Table 1 shows the relative amounts of concrete and steel required for the structure when analyzed by each of the three design theories. Table 1 discloses that the design made by the Conventional Theory requires 9.6 cubic yards more of concrete than the other designs. However, it requires 10.5 less steel than the Plastic Theory and 8.9 per cent less steel than the Ultimate Theory for the bents. In total steel required, the design by the Conventional Theory requires 1.5 per cent more steel than that by Ultimate Theory.

In terms of the total material weight, the design by the Conventional Theory requires 432,596 pounds of steel and concrete while the Plastic Theory requires 393,128 pounds and the Ultimate Theory 392,874 pounds. Thus, the Conventional Theory requires 39,468 pounds of material over that required by the Plastic Theory and 39,722 pounds more than the Ultimate Theory.

Table 2 shows the relative material costs for the three designs. As a result of these tabulations the design using the Ultimate Theory is the most economical in terms of weight and material costs. There is very little difference in the results obtained by the designs made using the Plastic and Ultimate Theories; however, either theory gives results that are much more economical in terms of weight than the design made by the Conventional Theory.

Rigid Frame Versus Hipped Plate Construction

Hipped plate construction, as applied to the structure selected for this analysis, requires much more material than was required for rigid frame construction. The concrete in the beams alone is greater than the total concrete required by any of rigid frame designs. Table 3 shows the material requirements for the hipped plate structure.

The data contained in Table 3 gives the material requirements for a structure of greater volume than that provided for in the rigid frame designs. Where the rigid frame construction provides the roof and a framework for the structure, the hipped plate structure contains structural members actually enclosing a portion of the sides. In this investigation it was assumed that masonry walls would be used to enclose the structural framework provided.

If the masonry walls of the structure are 12-inch thick brick walls and the window and door area is taken as 30 per cent of the entire wall area, a cost comparison can be made which will better show the economic merits of the two types of construction. Table 4 was compiled on this basis. Table 4 shows that rigid frame construction will be more conservative for the given structure from the standpoint of material costs. Another factor which is just as important in reinforced construction as the material costs is the weight transferred to the foundations. The rigid frame structure, composed of six two-hinged bents, will require twelve foundations; the hipped plate structure will require the equivalent of eight foundations. Table 5 shows the weight of structural framework transferred to the foundations of each type of construction.

Another item which must be considered for any type of construction is the quantity and cost of formwork required to support the structure while in the construction stage. The cost of roof joist forms for the rigid frame construction would be quite considerable since the joist size selected does not conform to the size of standard forms; however, if a number of similar structures were to be erected, as in a group of warehouses or airplane hangars, it would be much more economical to use special re-usable joist forms than to design for standard sections. Forming for the roof slabs of the hipped plate structure would be much simpler to construct, but heavier members would be required to support the greater slab weight. Due to the variable sections of the bents used in the rigid frame construction the forming would require additional time and material over that required for the hipped plate structure.

The method of erection would greatly influence an economic comparison between the two types of construction. It would be very profitable to construct the rigid frame roof joists in sections and erect them as prefabricated slabs. Such a procedure would be excellent where a number of identical structures are to be built. It would greatly reduce the amount of forming required for the rigid frame structure and thus, would present an additional economic factor in favor of this type of construction. Since the scope of this investigation covers only the relative material requirements for the structure, the discussion of formwork and method of erection will not be given further investigation.

Table 1. Quantities of Material Required for Rigid Frame Construction by the Conventional, Plastic and Ultimate Design Theories

		Conventional	Plastic	Ultimate
Concrete Required:	Roof Joists	46.0 c.y.	46.0 c.y.	46.0 c.y.
	Rigid Frames	53.0 c.y.	43.4 c.y.	43.4 c.y.
	Total	99.0 c.y.	89.4 c.y.	89.4 c.y.
Steel Required:	Roof Joists	10,230#	7,500#	7,500#
	Slab Temp. Steel	1,200#	1,200#	1,200#
	Bents 3 and 4	6,836#	7,440#	7,300#
	Stirrups	358#	536#	536#
	Ties	220#	220#	220#
	Bents 2 and 5	7,320#	8,080#	8,060#
	Stirrups	430#	830#	830#
	Ties	220#	220#	220#
	Bents 1 and 6	4,380#	4,460#	4,370#
	Stirrups	292#	528#	528#
	Ties	110#	110#	110#
	Total Steel:	31,596#	31,124#	30,904#

Table 2. Cost of Materials for Rigid Frame

Construction by the Conventional, Plastic and Ultimate Theories

	Total Concrete	Total Steel	Total Cost*
Conventional Theory	89.0 c.y.	31,596#	$4,325.00
Plastic Theory	89.4 c.y.	31,124#	4,140.00
Ultimate Theory	89.4 c.y.	30,904#	4,120.00

*The price of 3,000 psi. ready-mix concrete was taken as $15.00 per c.y. and the price of steel per lb. was taken as $0.09. (These costs do not include the forming or any labor costs.)

Table 3. Quantities of Material Required

for Hipped Plate Construction

Concrete:	4 - 30 ft. x 50 ft. plates	139.0 c.y.
	4 - 10 ft. x 50 ft. plates	46.3 c. y.
	4 - 2.16 ft. x 50 ft. diaphragms	159.2 c.y.
	8 - 2.16 ft. square columns	27.6 c.y.
	Total	372.1 c.y.
Steel:	Slabs 30 ft. x 50 ft.	14,380#
	10 ft. x 50 ft.	1,220#
	Diaphragms	5,980#
	Plates	10,420#
	Diagonal Tension	560#
	Columns	1,900#
	Total	34,460#

Table 4. Cost of Materials Required for Total Enclosure

	Concrete	Steel	Brick	Mortar	Total Cost*
Hipped Plate	372.0 c.y.	34,460#	101,000	33.0 c.y.	$13,260.00
Rigid Frame (Conventional Theory)	99.0 c.y.	31,596#	143,000	56.0 c.y.	10,945.00

*Does not include windows, doors, insulation, etc. The costs of materials was taken as follows:

Concrete - $15.00/c.y.
Steel - 0.09/#
Brick - 40.00/M
Mortar - 16.00/c.y.

Table 5. Weight of Concrete Structure Transferred to Foundations

	Total Weight	No. of Foundations	Wt./Foundation
Rigid Frame Conventional	432.6 kips	12.0	36.0 kips
Plastic	393.1 kips	12.0	32.8 kips
Ultimate	392.9 kips	12.0	32.75 kips
Hipped Plate	1554.5 kips	8.0	129.6 kips

CHAPTER IV

CONCLUSIONS

The material contained in this chapter is presented as a summary of conclusions drawn concerning the data presented in previous chapters and the Appendix. The material entails a comparison of the results obtained and a discussion of the results from the standpoint of economy. The conclusions, as given below, refer only to the quantities of concrete and steel required for the roof and structural framework of the structure analyzed. These conclusions may be stated as follows:

1. The design of the rigid frame structure using the Plastic and Ultimate Theories of reinforced concrete design will not materially reduce the cost of concrete and steel over that required by a design using the Conventional Theory if factors of safety giving comparable design stresses are used.
2. The total weight of the rigid frame structure when designed using the Conventional Theory will be at least 8.9 per cent greater than when designed using the Plastic or Ultimate Theory.
3. From the practical viewpoint, the results obtained from the rigid frame design using the Plastic and Ultimate Theories are identical.

4. The design made as a hipped plate structure will result in increased material costs over that required by rigid frame construction.

5. For the given base structure, hipped plate construction will produce an increase in the weight of structural materials of 350 - 400 per cent over rigid frame construction for identical loading conditions. However, the dimensions selected for the base structure were such that the type of construction was at a distinct disadvantage. In general, the economic efficiency of a hipped plate structure will be in proportion to the number of changes in slope in the roof slab.

CHAPTER V

RECOMMENDATIONS

This chapter contains a description of opinions formed by the writer during this investigation and suggestions for further research in this field. Due to the nature of the investigation the recommendations will be presented in two sections, one covering the more recent theories of reinforced design and the other covering hipped plate construction.

Plastic and Ultimate Theories of Reinforced Concrete Design

The scope and magnitude of reinforced concrete construction has, for a long time, been hampered by the relatively large dead weight involved. While, at the time of this investigation, reinforced concrete is more economical for structure of ordinary dimensions and height than steel, it cannot compete with steel for structures over 15 stories high in the U. S. A. Both the Plastic and Ultimate Theories present a logical approach to the concept of using the materials ultimate properties as a basis for design. The development of two theories, one from the empirical approach and the other from a theoretical approach, which agree so closely with each other would seem to warrant their acceptance by the engineering profession. The tendency among the majority of the engineering profession is to treat new methods and design approaches with suspicion and let some more enterprising individual prove, or disprove, in practice the technical considerations involved. This being the case, the writer recommends (1) that additional research be undertaken by universities and research organizations to establish the practicability of revising existing engineering specifications to include the Plastic and Ultimate Theories of reinforced concrete design; (2) that the concept of ultimate strength of materials should be given more publicity in order to acquaint the engineering profession with its existence and economic possibilities; (3) that the basic theory and nomenclature of the Plastic and Ultimate Theories of reinforced concrete design should be included with the Elastic Theory in the undergraduate curriculum of engineering schools to provide the profession with the technical knowledge and confidence required for their acceptance.

Hipped Plate Construction

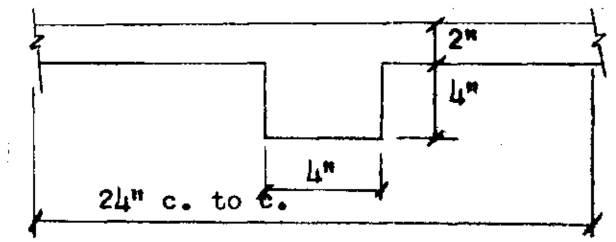
The economy of hipped plate construction was found to be dependent on the thickness of the roof slab. The roof slab, in turn, is dependent on the loadings and the span length. Therefore, by the method of analysis involved, a saw-tooth type of roof, or a roof made up of small slab lengths approximating a semi-circle, will be more economical for a structure of considerable proportions than a gable type as was used in this investigation. Thus, for any given span there will be some definite ratio of slab length to slab depth which would determine the most economical dimensions to be used. The writer recommends that an investigation be made with the derivation of a formula for determining the most economical proprotions for a given structure as its objective. The terms to be included in such a formula would be the overall span of the slabs, the span of the plates and the depth of the slabs or plates. Such a formula could be adapted to continuous construction and would provide a simple method of determining the relative economy of hipped plate construction with respect to other types under consideration.

The hipped plate method of construction as applied in this investigation was obviously at a disadvantage for the reason explained in the above paragraph. From its wide acceptance abroad it is evident that it presents certain economic advantages that were not apparent in this study. In order that the American engineering profession have access to the extensive material already available on hipped plate construction the writer recommends that English translations be made of that foreign literature which is most authoritative in this field. With such information available, together with our present knowledge of this field, it is probable that hipped plate construction will advantageously replace certain conventional type structures in this country.

A P P E N D I X

DESIGN CALCULATIONS--RIGID FRAME--CONVENTIONAL THEORY

Five rigid frames of variable section spaced twenty feet on centers were selected for the structures. The rigid frames will have a span length of 52' 0" (see Fig. 2). The A. C. I. Building Code Specifications were used in the design with allowable stresses of 3,000 psi. and 20,000 psi for concrete and steel respectively. The live and wind loading were each selected as 20 psf. Concrete joists were selected for the roof construction. They will have the dimensions shown in Fig. 21. While these joists are somewhat lighter than standard sections, they do not deflect excessively under loading or have the advantage of lightness.



Concrete Roof Joists

Figure 21

The design of the joist section is as follows:

Assume effective depth = 4.5 in.

Design load = 24.0 psF.

Shear: V = 4x4.5x7/8x90 = 1410 lbs. = WL

with L = 20 ft., W = 70.5 - 33 = 37.5 psF.

Bond: $V = \quad x\ 5/8 \times 7/8 \times 4.5 \times 300 = 2320 = 0.7\ WL$

with $L = 20$ ft., $W = 165. - 33 = 132.0$ psF

Positive M: (1 - #7 bar) $\quad A_c = 48.$ sq. in.

$A_s = \underline{\ 6.\ }$ sq. in.
54. sq. in.

$M = A_s\ j\ f_s d = 0.60 \times 0.59 \times 20{,}000 \times 4.5 = 46{,}350''\#$

$W = 77.0 - 33 = 44.00$ psF.

Negative M: (2 - #7 bars)

$M = 1.2 \times .845 \times 4.5 \times 20{,}000 - 91{,}200 - \dfrac{2WL^2 \times 12}{11}$

$W = 105 - 33 = 72.0$ psF.

For temperature steel, select #2 bars 12" o.c.

The deflection of the section was determined using the method developed by G. A. Maney.[1] This is

$$D - c\ \frac{L^2}{d}\ (e_c \neq e_s)$$

where $D =$ Maximum deflection in inches

$L =$ Span in inches

$d =$ Effective depth

$e_c =$ Unit deformation in concrete $= f_c/E_c$

$e_s =$ Unit deformation in steel $- f_s/E_s$

$c -$ Numerical constant dependent on loading conditions. ($c = 1/32$ for uniformly loaded fixed-end beams.)

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[1]G. Hool and W. S. Kinne, Structural Members and Connections, Second Edition, New York, McGraw-Hill Book Co., 1943, p. 94.

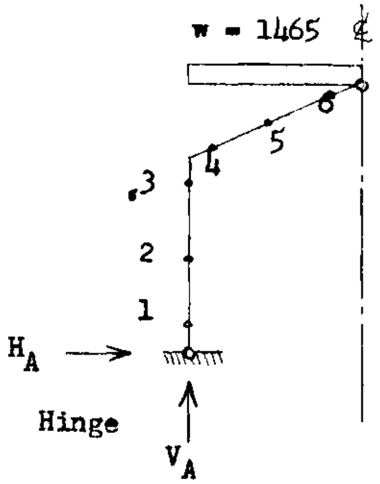
Solving the above expression, it was found that

$$D = 0.0668 \text{ in.}$$

Assuming the maximum allowable deflection to be L/300 or, in this case, 0.8 inches, it can be seen that the section will be adequate and will be used.

The required depths of arch section will now be investigated. Assuming the structure to be a constant-section, three-hinged arch, the approximate loading on the inclined member will be

Roof L. L. = 20 x 20 psF	= 400 plF. (approx.)
Roof D. L. = 20 x 33 psF	= 660
Section D. L.	= 225
Haunch D. L.	= 180
W	= 1.465 KlF

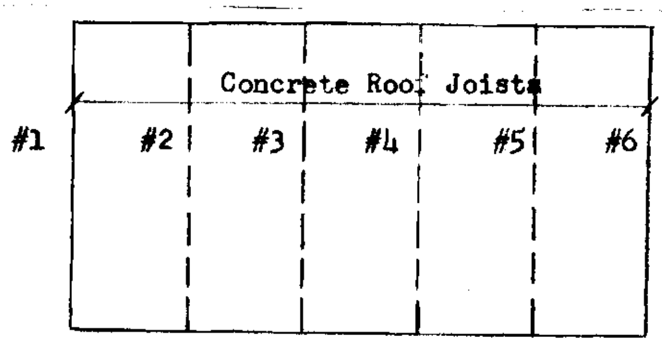


Three-hinged Bent Analysis

Figure 22

When the moments are found at the numbered points on the section, the corresponding section depths may be determined as shown in Table 6. The section, 1.5 feet wide, seems to fit the required steel areas very well. This width of section will be used in subsequent calculations. The depths at A, B and C will be 12 in., 30 in., and 6 in. respectively.

The A. C. I. Building Code contains in Section 701, factors to be used in approximating both the end moments and shears in continuous construction. In the design of the roof joists these factors were used to determine the maximum moment acting. Subsequently, in applying the factors to end shears it is found that the shear in the interior and first interior rigid frames will not have the same value. Referring to Fig. 23, the roof load acting on bents 1 and 6, 2 and 5, and 3 and 4, will be 0.5 WL, 1.075 WL and 1.0 WL respectively. Thus, it will be necessary to make separate analyses for the three loadings. Bents 1 and 6, being exterior bents, will have only one-half the section width, or 9 inches, of the other bents because the loadings taken by these bents will be only one-half that of the others.



Arrangement of Bents in the Structure

Figure 23

Table 6. Determination of Section Depths

Pt.	y	(Hy)M	Simple M	M	M/K	d	As	Bars	b
1	5.0	- 55.75		- 55.75	.262	12.5	3.08	2 #11	9.0
2	15.0	-167.5		-167.5	.71	22.0	5.25	1 #11 3 #10	15.5
3	25.0	-278.0		-278.0	1.18	29.0	6.6	1 #11 4 #11	18.5
4	32.5	-362.0	149.25	-212.75	.904	25.0	5.9	3 #11 1 #10	16.0
5	37.5	-417.5	372.0	- 45.5	.193	11.5	2.7	2 #11	9.0
6	42.5	-475.0	481.0	+ 6.0	.025	.50	.83	1 #10	--

Table 7. Properties and Hy Moments

Pt.	ds (ft.)	Y ft.	t ft.	I ft.	ds/I	M	Myds/I
1	10.0	5.0	1.25	0.245	40.8	5 H	1,020.H
2	10.0	15.0	1.75	0.67	14.9	15 H	3,350.H
3	10.0	25.0	2.25	1.42	7.07	25 H	4,400.H
4	10.0	32.5	2.167	1.27	7.88	32.5 H	8,300. H
5	10.0	37.5	1.50	.421	23.7	37.5 H	33,300.H
6	10.0	42.5	.833	.072	139.0	42.5 H	250,000.H

The bents 4 and 5 will be analyzed first. The analysis will be divided into six parts:

(1) Dead load.

(2) Live load full span.

(3) Live load left (or right) half span.

(4) Live load center half span.

(5) Moments produced by temperature change.

(6) Wind load on one-half span.

<u>Dead Load Analysis - Bents 3 and 4</u>

For the simple beam moments it is necessary to combine the moments due to the weight of the constant-section portion of the member BCD plus the moment due to the haunch weight.

For the constant-section member,

$$W = 0.5 \times 1.5 \times 150 = 112.5 \text{ plF.}$$

$$\text{plus the roof D. L.} = \underline{660.} \text{ plF.}$$

$$\text{Design } W = 772.5 \text{ plF.}$$

The simple beam moments produced by this loading are found by first solving for the left vertical reaction WL/2 and then determining the moments at points 4, 5 and 6 using the relationship

$$M_x = V_x - W_x{}^2/2$$

The bending moments produced by the weight of the haunching effect are determined by placing two symmetrical triangular loadings on a simple beam (Fig. 24) and solving for the moments at points 4, 5, and 6.

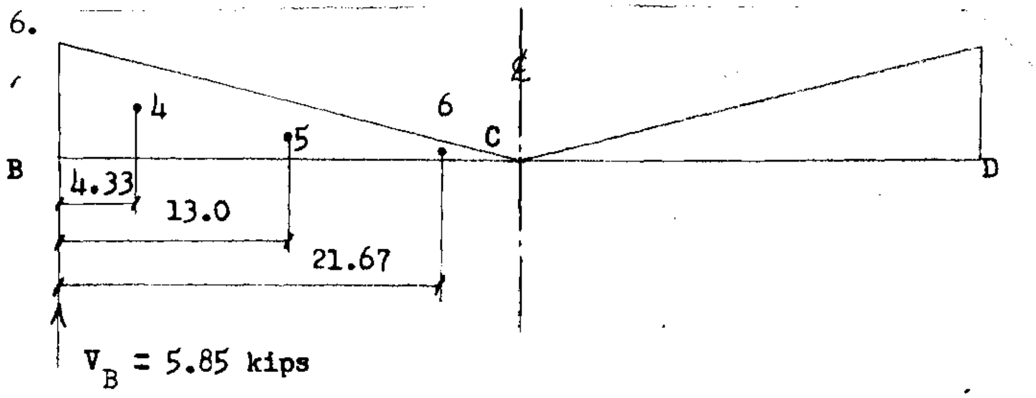


Figure 24

Effect of Haunch Weight

The combined dead load simple bending moments produced by the weight of the constant section member and the haunch are

$$M_4 = 78.0 + 21.4 = 99.4 \text{ ft. - kips}$$

$$M_5 = 201.0 + 44.2 = 245.2$$

$$M_6 = 258.0 + 50.3 = 308.3$$

$$M_C = 269.0 + 50.8 = 319.8$$

The tabulations in Tables 8 and 9 illustrate the method employed for determining the redundant force, H, and the final dead load moments for bents 3 and 4.

The live load full span moments are determined by the same procedure as the dead load moments. The load linear foot will be

$$W = 20' \times 20\# / 1 = 400 \text{ plF} = 0.4 \text{ klF.}$$

Table 8. Simple Beam Moments--Dead Load Full Span

Point	ds	y	ds/I	M	Mds/I	Myds/I
1	10.0	5.0	40.8			
2	10.	15.	14.9			
3	10.	25.	7.07			
4	10.	32.5	7.88	99.4	782.5	25,450.0
5	10.	37.5	23.7	245.2	5800.0	217,500.0
6	10.	42.5	139.0	308.3	42900.0	1,822,000.0

H = - 2,064,950 / 300,370 - 6.87 kips.

Table 9. Combined Dead Load Moments

Point	Simple M	HyM	Final M
1		- 34.3	- 34.3
2		-103.0	-103.0
3		-172.0	-172.0
4	99.4	-223.0	-123.6
5	245.2	-258.0	- 12.8
6	308.3	-292.0	≠ 16.0

The live load full span moments are determined by the same procedure as the dead load moments. The load/linear foot will be

$$w = 20' \times 20\#/1 = 400 \text{ plF} - 0.4 \text{ klf.}$$

Table 10. Simple Beam Moments--Live Load Full Span

Point	ds	y	ds/I	M	Mds/I	Myds/I
4	10.0	32.5	7.88	39.2	309.0	10,030.0
5	10.	37.5	23.7	101.4	2400.0	90,100.0
6	10.	42.5	139.0	131.3	18250.0	775,000.0
						875,130.0

H = 875,130/300,370 = - 2.91 kips.

Table 11. Combined Moments--Live Load Full Span

Point	Simple M	HyM	Final M
1		- 14.55	- 14.55
2		- 43.7	- 43.7
3		- 72.8	- 72.8
4	39.2	- 94.7	- 55.5
5	101.4	-109.1	- 7.7
6	131.3	-123.5	+ 7.8

The moments produced by the live load on one-half the span may be determined from the preceding case using the principle of symmetry and anti-symmetry. The H-force will be one-half of that for the preceding case and Table 12 may be compiled directly.

To determine the simple beam moments for the live load on the center-half span, the procedure is exactly the same as for the dead load of the constant section portion of the span. The live loading is placed on the center-half of the span and Equation is used to determine the simple beam moments at points 4, 5 and 6.

For the previous loading conditions the force, H (or Xa of Equation), could be solved for directly knowing the amount of the deflection, $\underline{d}_{aa}$, from Table 7 and the deflection $\underline{d}_{ao}$, from the summation of the simple beam loading condition. The effects of a temperature change on the structure must be calculated in a different manner because there is no actual loading on the structure. For this case, the deflection, $\underline{d}_{ao}$, is calculated directly from the relationship

$$\underline{d}_{ao} - C\ T\ L$$

where C = Coefficient of thermal expansion for concrete and steel equal to 0.000006'/° F.[2]

T = Temperature change, in degrees F.

L = Distance in feet of span, over which

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[2]H. Sutherland and R. C. Reese, Reinforced Concrete Design, Second Edition, New York, John Wiley and Sons, 1951, p. 44.

Table 12. Combined Moments--Live Load Left Half Span

Point	Simple M	HyM	Total M
1		- 7.26	
2		-21.75	
3		-36.30	
4	30.06	-54.4	- 17.14
5	67.2	-61.7	+ 12.8
6	75.0	-65.3	+ 13.3

Table 13. Simple Beam Moments--Live Load Center--Half Span

Point	y	ds/I	Simple M	Mds/I	Myds/I
4	32.5	7.88	22.5	177.5	5,770.0
5	37.5	23.7	67.6	1,605.0	60,400.0
6	42.5	139.0	97.75	13,600.0	578,000.0
					644,170.0

H = - 644,170/300,370 = -2.14 kips.

Table 14. Combined Moments--Live Load Center Half Span

Point	Simple M	HyM	Final M
1		-10.69	-10.49
2		-32.1	-32.1
3		-53.5	-53.5
4	22.5	-69.5	-47.0
5	67.6	-80.2	-12.6
6	97.75	-90.9	+ 6.85

the expansion or contraction will act.

For a temperature rise of 60° F.,

$$\underline{d}_{ao} = C\ T\ L = 0.000006 \times 60^{o} \times 52' = .\underline{01872}'$$

therefore, $$\int_{A}^{E} Myds/EI = 0.01872'$$

or, $$\frac{600,740\ H}{432 \times 103} + 0.01872' = 0$$

from which $$H = -\ 0.01872/1.39 = -\ 0.01348 \text{ kips.}$$

The bending moments produced by a temperature rise of 60° F. are shown in Table 14; the bending moments for a 60° F. temperature decrease will be of the same magnitude but of opposite sign.

The forces produced by the wind acting against the sides of the structure must be resisted by the individual bents (A.C.I. Building Code, 1951, Section 603a). It will be assumed that masonry walls are continuous between the bents and, therefore, the wind force taken by each bent will be in proportion to its individual stiffness.[3]

The dimensions of the interior bent for which the previous calculations have been made were determined by placing the uniform dead and live loads on a bent of constant section, and selecting the variable depths required from moment analysis. The A.C.I. Building Code, Section

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[3]Portland Cement Association, Continuity in Concrete Building Frames, Third Edition, 1951, pps. 45-53.

Table 15. Temperature Moments--60° F. Increase

Point	y	M
1	5.0	-.0673
2	15	-.202
3	25	-.337
4	32.5	-.438
5	37.5	-.505
6	42.5	-.572

701-6, states that, for conditions for which this design is favorable, the total shear at the first interior bents may be taken as 7.5 per cent greater than in the interior bents--and the total shear at all other bents will then be WL/2 (which was followed in the previous calculations). Since, for maximum construction economy, it is desirable that all bents be of similar proportions, the exterior bents will be estimated to have half the width (9 inches) of the interior bents already chosen, and the first interior bents (numbers 2 and 5) to have the same width (18 inches) as the remaining interior bents. As all members are identical in depth and length the individual stiffness of the bents will be in proportion to the width of section. That is

$$\text{Stiffness} = \underline{K} = \frac{EI}{L} = \frac{Ebd^3}{12L}$$

For bents 1 and 6, b = 9 in., $\underline{K}$ = 1.0 (relative).

For all other bents, b = 18 in., $\underline{K}$ = 2.0 (relative).

The wind force against the side of the structure was assumed to be 20 psF. The total wind force per bent will then be

$$20\# \times 100' = 2000 \text{ plF.}$$

The amount of this force transmitted to the individual bents is determined as follows:

$$\text{Wind Load} = \frac{2000}{\underline{K}\text{ Values}} \times \underline{K}\text{-value for bent}$$

For bents 1 and 6,

$$\text{Wind Load} = \frac{2000}{10} \times 1.0 = 200 \text{ plF.}$$

For the remaining bents,

$$\text{Wind Load} = \frac{2000}{10} \times 2.0 = 400 \text{ plF.}$$

With the wind loadings determined above, the final wind moments are calculated in a manner similar to the dead and the live load conditions.

Although the analysis is not yet finalized, the procedure used to obtain the required steel areas will be briefly described. It is believed that this will enable the reader to follow the subsequent tabulations more easily. The method used to obtain the required amount of steel at each section is as follows:

1. The moments produced at each point by the various loading conditions previously described are tabulated.
2. The combination of loading conditions producing the maximum moment at each point is determined. At some points it was found that two maximum conditions exist.
3. The maximum value of the thrust produced by the combination of loading conditions which result in the maximum bending moment at that point is obtained.
4. Tables contained in The Reinforced Concrete Design Handbook were referred to in computing the steel areas. The method of equivalent eccentric load was used, when applicable.
   The method consists of computing the equivalent eccentricity, E, of the thrust, N, which would produce the same effect on

Table 16. Simple Beam Moments--Wind from Left

Point	y	ds/I	M	Mds/I	Myds/I
1	5.0	40.8	43.5	1778.0	8,900.0
2	15.	14.9	89.2	1330.	20,000.
3	25.	7.07	98.25	695.	17,340.
4	32.5	7.88	81.0	638.	20,700.
5	37.5	23.7	55.7	1320.	52,300.
6	42.5	139.0	28.3	3930.	167,200.
				=	286,440.0

H = 286,440/600,740 = - 0.476 ft. kips

Table 17. Combined Moments--Wind from Left

Point	Simple M	HyM	Final M
1	43.5	- 2.38	+ 41.12
2	89.2	- 7.13	+ 82.07
3	98.25	- 11.19	+ 86.35
4	81.0	- 15.45	+ 65.55
5	55.7	- 17.83	+ 37.87
6	28.3	- 20.2	+ 8.1
6'	0.0	- 20.2	- 20.2
5'	0.0	- 17.83	- 17.83
4'	0.0	- 15.45	- 15.45
3'	0.0	- 11.19	- 11.9
2'	0.0	- 7.13	- 7.13
1'	0.0	- 2.38	- 2.38

the section as the bending moment and thrust combined. The properties K, dependent on the strength of the materials used, and F, dependent on the section dimensions, are obtained from Tables 1 and 4 respectively. Should the value, NE, exceed the value, KF, no compression steel is required. If no compressive reinforcing is required at the section of the tensile steel area is computed from the formula

$$A_s = \frac{NE}{adi}$$

where d = effective depth of section

a and i = constants found in Tables 1, 3 and 10 in the Handbook.

When compression steel is required, its area is computed using the relationship

$$A'_s = \frac{NE-KF}{cd}$$

where c = section constant given in Table 7 of the Handbook.

In those cases where the use of the equivalent eccentric load does not apply (that is, for small eccentricities) the section was designed using conventional column formulas.

The thrust and shear on any section are computed as described in Chapter II. A tabulation of these forces for bents 3 and 4 is shown in

Table 18. Thrust and Shear--Bents 3 and 4

Pt.	V	VsinΘ	HcosΘ	N	HsinΘ	VcosΘ	VN
a. Dead Load*							
1	36.05	36.05		36.05	6.87		-6.87
2	31.1	31.1		31.1	6.87		-6.87
3	28.45	28.45		28.45	6.87		-6.87
4	20.0	10.0	5.95	15.95	3.44	17.4	13.96
5	10.82	5.41	5.95	11.36	3.44	9.38	5.94
6	3.35	1.67	5.95	7.62	3.44	2.98	0.46
b. Live Load Full**							
1	10.4	10.4		10.4	2.91		2.91
2	10.4	10.4		10.4	2.91		2.91
3	10.4	10.4		10.4	2.91		2.91
4	8.54	4.27	2.52	6.79	1.45	7.4	5.95
5	5.2	2.6	2.52	5.12	1.45	4.5	3.05
6	1.7	.85	2.52	3.37	1.45	.737	1.11

*V_L = 25.85 kips

H = - 6.87 kips

V_{TL} = 37.6 kips

sinΘ = 0.5

cosΘ = 0.866

**V_L = 10.4 kips

H = - 2.91 kips

Table 19. Maximum Moments--Bents 3 and 4

Pt.	D.L. Full	Full	Live Load Left	Live Load Right	Center	Temp. +60°	-60°	Wind Left	Wind Right	Max. Comb.
1	-34.3	-14.55	-7.26	-7.26	-10.69	-.067	+.067	+41.72	-2.38	(-51.30 (+ 7.42
2	-103.0	-43.7	-21.75	-21.75	-32.1	-.202	+.202	+82.07	-7.13	-154.00
3	-172.0	-72.8	-36.30	-36.30	-53.5	-.336	+.336	+86.32	-11.19	-257.04
4	-123.6	-55.5	-17.14	-35.2	-47.0	-.436	+.436	+65.55	-15.45	-195.00
5	- 12.8	- 7.7	+12.8	- 20.4	-12.6	-.505	+.505	+37.87	-17.83	(- 43.74 (+ 38.38
6	+ 16.0	+ 7.8	+13.3	- 5.7	+ 6.5	-.572	+.572	+ 8.1	-20.2	+37.97) -10.47)

Table 20. Elastic Theory--Steel Required--Bents 3 and 4

Pt.	M l-k	N	e = M/N	t	d	E	NE	F	KF	C	i	Top as $\frac{M}{adi}$	Top Bars	Bot. as $\frac{NE-KF}{cd}$	Bot. Bars
1	-51.30	52.53	11.75	15	13	1.44	75.5	2.54	59.8	.528	2.81	1.43	1#11	2.29	1#11, 1#10
	+7.42	28,2	3.15	15	13	.72	20.3	.254	59.8	--	--	--	--	1.09	1 #10
2	-154.	47.58	39.1	21	19	3.96	188.	.541	127.5	.68	1.54	4.48	3#11	4.58	3 #10
3	-257.	44.93	67.5	27	25	6.75	303.	.89	210.	.75	1.4	6.0	4#11	4.95	2 #11, 2#10
4	-19.5	28.93	83.7	26	24	7.45	226.	.854	201.5	.75	1.28	5.1	(1#11 (3#10	.885	1 #10
5	- 43.74	18.23	28.8	18	16	2.98	57.4	.384	90.6	.60	1.64	1.52	1 #11	--	--
	+38.38	7.42	62.2	18	16	5.76	42.7	.384	90.6	.60	1.25	--	--	(1.48)	1 #11
6	+10.47	14.5	8.65	10	8	.97	14.3	.096	22.6	.31	1.91	.805	1 #10	--	--
	-37.97	3.71	12.3	10	8	10.5	38.9	.096	22.6	.31	1.06	(3.31)	1 #11 2 #10	(7.2)	3#11, 2#10

Table 21. Shear and Bond--Bents 3 and 4

Pt.	V. Max.	G	v psi.	Bond u	Stirrups
1	10.25	.208	49.3	62.5	#4 hoops 24" c/c
2	10.25	.299	34.3	20.2	do
3	10.25	.349	27.6	12.72	do
4	36.77	.378	97.0	72.0	See below
5	25.37	.252	100.0	131.	do
6	18.97	.126	150.0	57.5	do

For stirrups on inclined member:

#3 Hoops:

$$\text{Max } 1/s = \frac{80\text{x}18}{4400} = .327$$

$$\text{Min } 1/s = \frac{7\text{x}18}{4400} = \frac{.029}{97}$$

$$N = 150\ (.356) = 54$$

$$\text{Index} = \frac{37.5}{.297} = 126$$

Starting at peak of gable use:

14@ 3",
21@ 4",
10@ 6",
5@ 8",
5@12",

in each direction.

Table 18.

The results of previous tabulations for moments under various loadings are tabulated and the maximum moments at each section are determined. This is shown in Table 19. The maximum moments for the points on the right half of the arch will be identical to those of corresponding points shown in Table 18.

The distribution of the dead and live loads from the roof joists will vary for the exterior and the two interior bents. For bents 2 and 5 the roof dead load will increase to

$$1.075 \times .660 \text{ kips} - 0.709 \text{ klF.}$$

Adding the weight of the constant-section portion of the bent to the above loading will give

$$0.150 \neq 0.709 = 0.859 \text{ klF.}$$

Computing the simple beam bending moments for this loading, adding the moments resulting from the haunch weight, and adding to the result the moments due to the horizontal forces at the hinges will give, as before, the true dead load moments.

Similarly, the live loading acting on bents 2 and 5 will increase to

$$1.075 \times .400 = 0.430 \text{ klF.}$$

Computing the true live load moments for the four conditions of live loading will give the moments required for these bents.

Since these calculations (and also those for bents 1 and 6) are so similar to those shown for bents 3 and 4, they will not be shown here. Instead, only the final results will be given. These are shown in the tabulations which follow.

Table 22. Maximum Moments in Bents 2 and 5

Point	Dead Load	Live Load Full	Live Load Left	Live Load Right	Live Load Center	Temp. +60°	Temp. -60°	Wind Left	Wind Right	Max Comb.
1	-36.1	-15.85	-7.8	-7.8	-11.48	-.0673	+.0673	+41.12	-2.38	(-54.40 (+ 5.08
2	-108.2	-46.1	-23.4	-23.4	-34.5	-.202	+.202	+82.07	-7.13	-161.63
3	-180.5	-78.2	-39.0	-39.0	-57.5	-.337	+.337	-+86.35	-11.9	-270.44
4	-130.6	-59.6	-18.4	-18.4	-50.6	-.438	-.438	+65.55	-15.45	-208.47
5	- 12.8	-8.26	+13.75	+13.75	-13.55	-.505	-.505	+37.87	-17.83	(-52.5 (+39.3
6	+ 17.3	+8.38	+14.3	- 6.1	+ 7.37	-.572	+.572	+8.1	-20.2	(- 9.57 (+40.27

Table 23. Thrust and Shear--Bents 2 and 5

Point Condition	1 N	1 V	2 N	2 V	3 N	3 V	4 N	4 V	5 N	5 V	6 N	6 V
D. L.	38.8	7.22	33.9	7.22	31.2	7.22	17.91	21.57	12.35	14.50	9.31	9.08
L. L. Full	9.31	3.13	9.31	3.13	9.31	3.13	7.36	9.62	4.58	4.78	2.72	1.57
L. L. Left	8.05	1.56	8.05	1.56	8.03	1.56	4.62	6.43	2.71	2.71	.885	.032
L. L. Right	2.79	1.56	2.79	1.56	2.79	1.56	2.75	3.2	2.75	2.75	2.75	3.2
L. L. Center	5.59	2.3	5.59	2.3	5.59	2.3	4.78	5.98	4.78	4.78	2.92	5.98
Wind Left	8.39	17.2	8.39	12.9	8.39	8.6	8.82	11.45	7.16	7.16	5.08	7.82
Wind Right	8.39	.51	8.39	.51	8.39	.51	4.63	7.5	4.63	4.63	4.63	7.5

Table 24. Elastic Theory--Steel Required--Bents 2 and 5

Pt.	M	N	e	t	d	E	NE	F	KF	C	i	Top As $\frac{M}{adi}$	Top Bars	Bot. As	Bot. Bars
1	-54.40	56.50	11.52	15	13	1.42	80.4	.254	59.8	.54	2.85	1.5	1 #11	4.36	2 #11, 1 #10
	+ 5.08	30.41	2.01	15	13	.541	16.42	.253	59.8	.54	--	--	--	(1.11)	1 #10
2	-161.63	51.60	36.5	21	19	3.75	193.5	.541	127.5	.67	1.54	4.58	3 #11	5.18	1 #11, 3 #10
3	-270.44	48.90	66.5	27	25	6.5	318.	.89	210.	.75	1.40	6.3	(1#11 (4#10	5.76	4 #11
4	-208.47	29.90	83.7	26	24	7.88	236.	.854	201.5	.75	1.28	5.33	1 #11) 3 #10)	1.92	1 #10, 1 #11
5	- 52.5	19.73	31.9	18	16	3.24	64.	.384	90.6	.60	1.57	1.77	2#10	--	--
	+ 39.3	7.90	59.7	18	16	5.45	43.1	.384	90.6	.60	1.27	--	--	(1.47)	1 #11
6	- 9.57	13.94	8.25	10	8	.935	13.	.096	22.6	.31	2.55	.444	1 #10	--	1 #11
	+ 40.27	5.112	94.5	10	8	8.02	41.0	.096	22.6	.31	1.08	(3.23)	1 #10 2 #11	7.42	4 #11* 1 #10

*Place steel in 2 rows

Table 25. Shear and Bond--Bents 2 and 5

Pt.	V Max.	G*	v psi.	Bond u	Stirrups**
1	10.86	.243	44.7	49.4	#2 Hoops, 24" c/c
2	10.86	.299	36.3	19.2	do
3	10.86	.394	27.6	11.65	do
4	38.69	.378	98.7	66.2	See below
5	27.98	.284	98.5	153.0	do
6	22.58	.131	172.0	133.0	do

*Obtained from Table 14, Handbook.

**ACI Code, Section 706(a).

For stirrups on inclined member use #3 hoops:

29 @ 3"

15 @ 4"

9 @ 6"

4 @ 8"

9 @ 12"

starting at peak of gable and extending in each direction.

Table 26. Maximum Moments--Bents 1 and 6

Point	Dead Load	Live Load Full	Live Load Left	Live Load Right	Live Load Center	Temp. +60°	Temp. -60°	Wind Left	Wind Right	Max. Comb.
1	-17.15	-7.27	-3.63	-3.63	-5.35	- .134	+.134	+20.86	-1.19	-25.74, +3.84
2	-51.5	-21.9	-10.87	-10.87	-16.05	-.404	+.404	+41.04	-3.57	-77.37
3	-86.	-36.4	-18.15	-18.15	-26.75	-.772	+.772	-+43.18	-5.95	-129.05
4	-61.8	-27.75	- 8.57	- 8.57	-23.5	-.876	+.876	+32.78	-7.73	- 98.16
5	- 6.4	- 3.85	+ 6.4	+ 6.4	-6.3	-1.10	+1.10	+18.94	-8.9	- 26.6, + 18.94
6	+ 8.0	-/3.9	+ 6.65	+ 6.65	+3.25	-1.144	+1.144	+ 4.05	-10.1	- 5.59, + 18.70

Table 27. Thrusts and Shears--Bents 1 and 6

Point	1		2		3		4		5		6	
Condition	N	V	N	V	N	V	N	V	N	V	N	V
D. L.	18.02	3.44	15.5	3.44	14.2	3.44	7.98	10.42	5.68	6.41	3.81	3.21
L. L. Full	4.38	1.45	4.38	1.45	4.38	1.45	3.43	4.48	2.13	2.23	1.27	.73
L. L. Left	3.75	.725	3.75	.725	3.75	.725	2.15	2.99	1.28	1.49	.412	-.01
L. L. Right	1.3	.725	1.3	.725	1.3	.725	1.28	1.49	1.28	1.49	1.28	1.49
L. L. Center	2.6	1.07	2.6	1.07	2.6	1.07	2.22	2.79	2.22	2.79	1.36	2.79
Wind Left	-3.9	-8.0	-3.9	-6.0	-3.9	-4.0	-4.1	-5.33	-3.25	-4.47	-2.37	-3.64
Wind Right	3.9	.238	3.9	.238	3.9	.238	2.15	3.49	2.15	3.49	2.15	3.49

Table 28. Elastic Theory--Steel Required--Bents 1 and 6

Pt.	M	N	e = $\frac{M}{N}$	t	d	E	NE	F	KF	c	i	Top As	Top Bars	Bot. As	Bot. Bars
1	-25.74	26.30	11.71	15	13	1.435	37.7	.126	29.8	.54	2.81	.718	1 #10	1.14	1 #10
	+ 3.84	14.1	3.18	15	13	.725	10.2	.126	29.8	.54	--	--	--	(.55)	1 #10
2	- 77.37	23.79	39.	21	19	3.96	94.2	.271	64.	.69	1.53	2.25	2 #10	2.3	2 #10
3	-129.05	22.47	69.2	27	25	6.7	151.	.469	111.	.75	1.38	3.04	2 #11	2.13	2 #10
4	- 98.16	14.47	81.5	26	24	7.7	111.5	.432	102.	.75	1.31	2.47	2 #10	.527	1 #10
5	- 26.6	9.12	35.1	18	16	3.51	32.	.192	45.4	.60	1.50	.926	2 #11	--	--
	+ 18.9	3.21	71.0	18	16	6.48	20.8	.192	45.4	.60	1.26	--	--	(2.53)	1 #10
6	- 5.59	7.25	9.25	10	8	.946	6.78	.048	11.32	.31	2.25	.283	1 #10	--	--
	+ 18.70	1.85	123.	10	8	10.5	19.4	.048	11.32	.31	1.06	(1.53)	1 #11	(3.13)	2 #11

Table 29. Shear and Bond--Bents 1 and 6

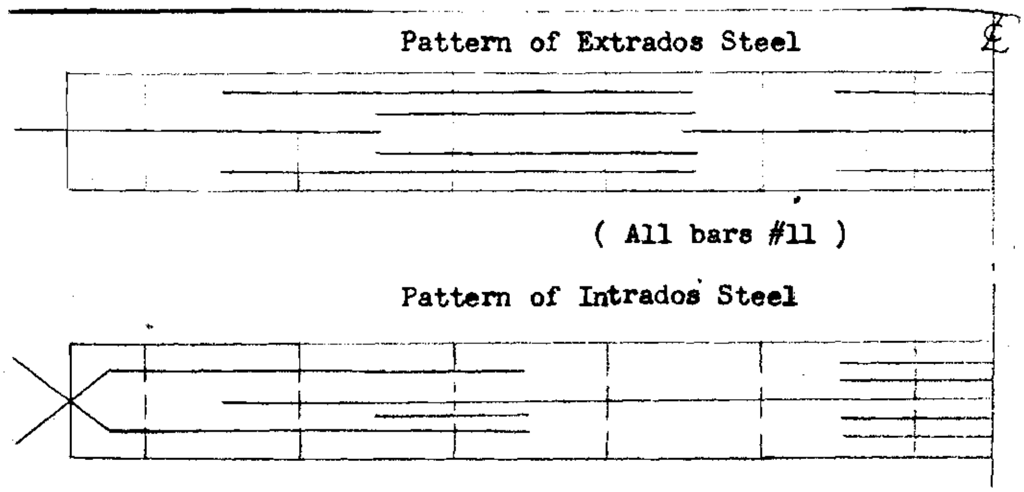
Pt.	V Max.	G	psi	Bond u	Stirrups
1	5.13	.102	50.4	48.	#4 Hoops, 24" c/c
2	5.13	.150	34.2	17.35	do
3	5.13	.197	26.0	16.3	do
4	18.34	.189	97.	60.8	See below
5	12.69	.126	101.	96.0	do
6	9.49	.063	150.0	91.2	do

For stirrups on inclined member:

Use #3 Hoops,

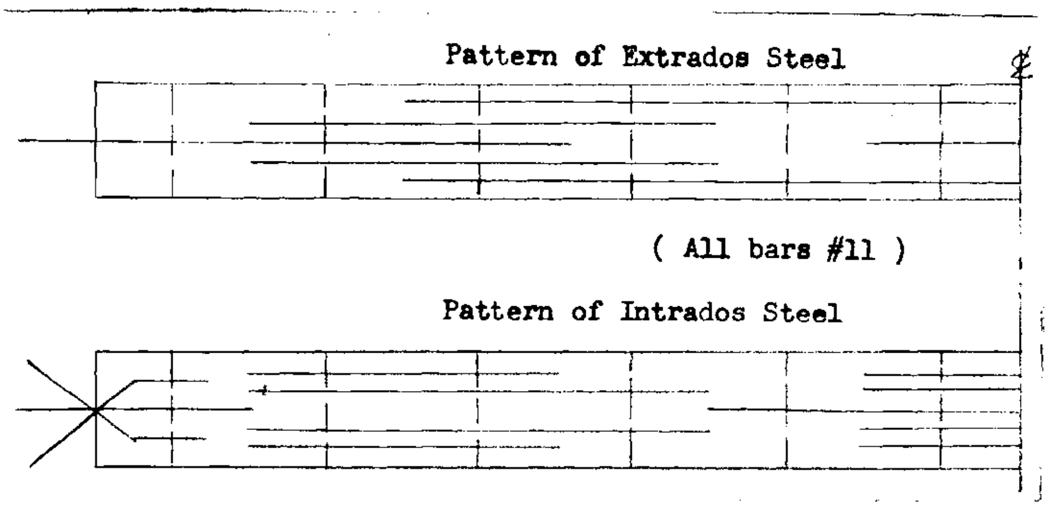
14 @ 3"
21 @ 4"
10 @ 6"
5 @ 8"
5 @12"

starting at peak of gable and extending in each direction.



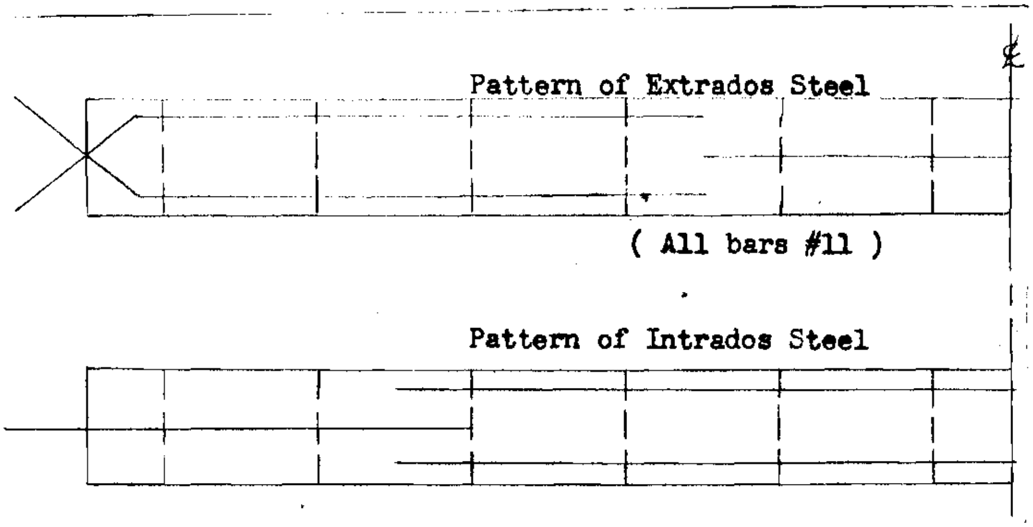
Steel in Bents 3 and 4

Figure 25



Steel in Bents 2 and 5

Figure 26



Steel in Bents 1 and 6

Figure 30

DESIGN CALCULATIONS--RIGID FRAME--PLASTIC THEORY

Five rigid frames of variable section spaced twenty feet on centers were selected for the structure. The allowable stresses used in this design were 50,000 psi. and 3,000 psi. for steel and concrete respectively. A factor of safety of 2.5 was applied to both the steel and concrete stresses making the design stresses equal to 20,000 psi. and 1,200 psi. for steel and concrete. The live and wind loadings are the same as those used in the design by the Conventional Theory.

Concrete roof joists of the same size and section illustrated in Figure 21 were used in this design. The design of the roof joists is as follows:

Projected loadings - roof L. L. = 14.0 psF.

wind load = 10.0 psF.

Design load 24.0 PsF.

$Mn = WL^2/11 = 10,500$ in.-lb.

$Mp = WL^2/16 = 7,200$ in.-lb.

Shear: $V = 4 \times 4.5 \times 7/8 \times 90 = 1410 = WL$

$L = 20$ ft., $W = 70.5 - 33 = 37.5$ psF.

Bend: $V = \quad \times 3/4 \times 7/8 \times 4.5 \times 300 = 2160 = 0.7\ WL$

$L = 20$ ft., $W = 154 - 33 = 121.0$ psF.

Positive M: $P = 0.44/24.0 = 0.0171$
(1- #6)

$M = 20,000/1,200 = 16.65$

$C = 4.5\ (1 - 0.0171 \times 16.6512) = 3.87$ in.

$M = CAsfs = 34{,}200$ in.-lb. $= 600\ W$

$L = 20$ ft. $W = 57.0 - 33. = 24.0$ psF.

Negative M: $P = 0.88/24.0 = 0.0367$
(2 - #6)

$C = 3.13$ in.

$M = 3.13 \times 0.88 \times 20{,}000 = 55{,}000$ in.-lb. $= 865\ W$

$W = 64.5 - 33.0 = 31.5$ psF.

The section is adequate and will be used.

The maximum design moments for the bent when analyzed as a constant-section three-hinged arch were found in the calculations for the Conventional Theory design (Table 6). These moments will again be used to determine the required section depths using the Plastic Theory. Equation   is used to determine the section depths; the form of the equation is as follows:

$$d = (3M/b\ f'c)^{1/2}$$

The calculations for the section depths are shown in Table 30. From Table 30 the section depths at A, B and C (Figure   ) were chosen as 10 in., 24 in. and 6 in. respectively.

To compute the dead load simple bending moments, the variable section member is divided into two portions: a constant section portion and a section consisting of the haunching weight. The design loading of the constant-section portion will be uniform and will consist of the weight of the member plus the roof dead load or, weight of constant section inclined member = 112.5 plF.

plus roof dead loading = 660.0 plF.

Design load, w = 772.5 plF.

The weight of the haunching effect will consist of two triangular shaped sections symmetrically placed on both sides of the inclined member. The dead load simple bending moments for both the constant and variable loadings are determined for points 4, 5 and 6 on the bent and added to obtain the total bending moment at each point. These moments are then tabulated as shown in Table 32 to obtain the deformation $\underline{d}_{ao}$, produced by the external loadings.

Similarly, for the live loading on the full span

$$w = 20 \text{ psF} \times 20 \text{ ft.} = 0.4 \text{ klF.}$$

In Table 35, as in the similar analysis by the Conventional Theory, the principles of symmetry and anti-symmetry are employed to determine the horizontal force, H, for the loading condition for the live load on one-half the structure. For this condition, the horizontal force will be one-half that for full live loading or, -1.82 kips. The combined moments for this condition may be then obtained directly.

The principles of symmetry and anti-symmetry cannot be applied to the condition of live load on the center half of span as directly as in the preceding case, and for this case, the simpler bending moments are calculated in the same manner as for the dead load condition.

Table 30. Required Section Depths

Point	M ft. kips	3 M ft-kips	3 M in.-lbs.	b f'c	$\frac{3M}{bf'c} = d^2$	d
1	-55.75	-167.5	-2,010,000.	21,600.	93.2	9.66
2	-167.5	-502.5	-6,030,000.		279.5	16.72
3	-278.0	-844.0	-10,130,000.		470.0	21.7
4	-212.75	-638.25	-7,560,000.		350.0	18.7
5	- 45.5	-136.5	-1,638,000.		75.9	8.72
6	+ 6.0	+ 18.0	+ 216,000.		10.0	3.16

Table 31. Section Properties and Hy Moments--Bents 3 and 4

Pt.	ds	y	t	I	ds/I	M	Mds/I	Myds/I
1	10.0	5.0	0.958	0.110	91.0	5H	454.H	2,270.H
2	10.0	15.0	1.375	0.325	30.8	15H	464.H	6,950.H
3	10.0	25.0	1.79	0.716	13.98	25H	349.H	8,720.H
4	10.0	32.5	1.75	0.668	14.97	32.5H	486.H	15,750.H
5	10.0	37.5	1.25	0.224	41.0	37.5H	1540.H	57,600.H
6	10.0	42.5	0.75	0.0529	189.0	42.5H	8030.H	341,000.H
							=	432,300.H
								x 2
								864,600.H

Table 32. Simple Beam Moments--Dead Load Full Span

Point	ds	y	ds/I	M	Mds/I	Myds/I
1	10.0	5.0	91.0			
2	10.0	15.0	30.8			
3	10.0	25.0	13.98			
4	10.0	32.5	14.97	94.05	1405.0	45,700.0
5	10.0	37.5	41.0	234.1	9600.0	360,000.0
6	10.0	42.5	189.0	295.8	55,800.0	2,370,000.0
						2,775,000.0

H = 2,775,000 /432,300 = 6.43 kips

Table 33. Combined Dead Load Moments

Point	Simple M	HyM	Total M
1		- 32.1	- 32.1
2		- 96.5	- 96.5
3		-161.0	-161.0
4	94.05	-114.95	-114.95
5	234.1	- 6.9	- 6.9
6	295.8	≠ 22.8	≠ 22.8

Table 34. Simple Beam Moments--Live Load Full Span

Point	ds	y	ds/I	M	Mds/I	Myds/I
4	10.0	32.5	14.97	39.2	587.0	19,050.0
5	10.0	37.5	41.0	101.4	4160.0	156,000.0
6	10.0	42.5	189.0	131.3	24800.0	1,054,000.0
						1,229,050.0

H = 1,229,050 / 432,500 = - 2.84 kips

Table 35. Combined Moments--Live Load Full Span

Point	Simple M	HyM	M
1		- 14.2	- 14.2
2		- 42.6	- 42.6
3		- 71.0	- 71.0
4	39.2	- 92.4	- 53.2
5	101.4	-106.2	- 5.8
6	131.3	-120.5	≠ 10.8

Table 36. Combined Moments--Live Load Half Span

Point	Simple M	HyM	Total M
1		- 7.06	- 7.06
2		-20.2	-20.2
3		-35.3	-35.3
4	30.06	-45.9	-15.84
5	67.20	-53.0	+14.2
6	75.0	-60.0	+15.0

Table 37. Simple Beam Moments--Live Load Center Half Span

Point	y	ds/I	M	Mds/I	Myds/I
4	32.5	14.97	22.5	337.0	10,950.0
5	37.5	41.0	67.6	2,760,0	103,600.0
6	42.5	189.0	97.75	18,500.0	787,500.0
					902,050.0

H = - 902,050/432,300 = - 2.09 kips.

Table 38. Combined Moments--Live Load Center--Half Span

Point	Simple M	HyM	M
1		- 10.45	- 10.45
2		- 31.3	- 31.3
3		- 52.2	- 52.2
4	22.5	- 67.8	- 45.30
5	67.6	- 78.4	- 10.8
6	97.75	- 88.7	+ 9.05

For moments induced by a temperature differential of 60° F.,

$$\underline{d}_{ao} = CTL = 0.000006 \times 60^\circ \times 52' = 0.01872'$$

and

$$\frac{864,600\ H}{3 \times 10^3 \times 144} \neq 0.01872' = 0$$

Therefore, $H = 18.72/1998.0 = -0.00938$ kips.

The bending moments produced by a temperature rise of 60° F. are shown in Table 39. The previous calculations have been made for the interior bents, 3 and 4. The procedure, as in the design by the Conventional Theory, has been to divide the six bents into three groups according to their loading. A description of the method is given in detail in the previous design calculations. The moments produced by the wind load acting against the left side of the structure are given in Tables 40 and 41.

Values of thrust and shear at the different points are obtained as described in Chapter II, and a tabulation of the resulting thrust and shears under the various loading conditions is shown in Table 42.

First Interior Bents (2 and 5)

The roof dead load will increase to 1.075 x .660 = 0.709 **klF**, while the roof live load will increase to 1.075 x .400 - .430 klF. The moments due to the haunching effect will be the same as for bents 3 and 4. Then, the dead load moments at points 4, 5 and 6 will be as follows:

$$M_4 = 82.8 + 16.05 = 98.85 \text{ '-K}$$

$$M_5 = 213.5 + 33.1 = 246.6$$

$$M_6 = 274.0 + 37.8 = 311.8$$

$$M_c = 285.5 + 38.1 = 326.6$$

To obtain the true Dead Load moments, the above moments are tabulated and the Summation Myds/EI is obtained as follows:

Pt.	ds	y	ds/I	M	Mds/I	Mdsy/I
4	10.	32.5	14.97	98.85	1,478.	48,100.
5	10.	37.5	41.0	246.6	10,100.	379,000.
6	10.	42.5	189.0	311.8	58,800.	2,500,000.
					SUM. =	2,927,100.

$$H = -\frac{2,927,100}{432,300} = -6.77 \text{ K.}$$

The true Dead Load moments are then as follows:

Pt.	Simple M	(Hy)M	M
1		- 34.85	- 34.85
2		- 101.6	- 101.6
3		- 169.5	- 169.5
4	98.85	- 220.0	- 121.15
5	246.6	- 254.0	- 7.4
6	311.8	- 288.0	+ 23.8

Table 39. Combined Moments--60° F. Temperature Rise

Point	y	M
1	5.0	- .0673
2	15.0	- .202
3	25.0	- .337
4	32.5	- .438
5	37.5	- .505
6	42.5	- .572

Table 40. Simple Beam Moments--Wind from Left

Point	y	ds/I	M	Mds/I	Myds/I
1	5.0	91.0	43.5	3970.0	19,850.0
2	15.0	30.8	89.2	2760.0	41,400.
3	25.0	13.98	98.25	1375.0	34,350.
4	32.5	14.97	81.0	1209.0	39,200.
5	37.5	41.0	55.7	2278.0	85,250.
6	42.5	189.0	28.3	5360.0	227,500.
					447,550.0

Table 41. Combined Moments—Wind from Left

Point	Simple M	HyM	M
1	43.5	- 2.575	40.925
2	89.2	- 7.72	81.48
3	98.25	- 12.88	85.37
4	81.0	- 16.70	74.3
5	55.7	- 19.3	36.4
6	28.3	- 21.9	6.4
6'		- 21.9	- 21.9
5'		- 19.3	- 19.3
4'		- 16.70	- 16.7
3'		- 12.88	- 12.88
2'		- 7.72	- 7.72
1'		- 2.575	- 2.575

Table 42. Thrusts and Shears--Bents 3 and 4

Point	1		2		3		4		5		6	
Condition	N	V	N	V	N	V	N	V	N	V	N	V
D. L.	29.3	6.43	26.0	6.43	23.4	6.43	13.66	17.24	10.06	10.93	6.94	5.60
L. L. Full	10.4	2.84	10.4	2.84	10.4	2.84	6.79	8.92	4.84	5.55	3.11	2.55
L. L. Left	7.8	1.41	7.8	1.41	7.8	1.41	4.26	5.96	3.52	2.95	0.78	0.045
L. L. Right	2.6	1.41	2.6	1.41	2.6	1.41	2.52	2.95	2.52	2.95	2.52	2.95
L. L. Center	5.2	2.07	5.2	2.07	5.2	2.07	4.4	5.54	4.4	5.54	2.66	2.54
Wind Left	-7.8	-15.0	-7.8	-11.0	-7.8	-7.0	-8.38	-11.25	-6.68	-5.25	-4.55	-7.26
Wind Right	7.8	0.515	7.8	0.515	7.8	0.515	4.35	6.97	4.35	6.97	4.35	6.97

Table 43. Maximum Combined Moments--Bents 3 and 4

Point	Dead Load	Live Load Full	Live Load Left	Live Load Right	Live Load Center	Temp. +60°	Temp. -60°	Wind Left	Wind Right	Max. Comb.
1	-32.1	-14.2	-7.06	-7.06	-10.45	-.042	.042	40.93	-2.57	(-48.87 (+8.83
2	-96.5	-42.6	-21.3	-21.3	-31.3	-.141	.141	81.48	-7.72	-146.82
3	-161.0	-71.0	-35.3	-35.3	-52.2	-.234	.234	85.37	-12.88	-245.08
4	-114.9	-53.2	-15.84	-33.9	-45.3	-.305	.305	74.3	-16.7	-185.15
5	-6.9	-5.8	+14.2	-19.0	-10.8	-.351	.351	36.4	-19.3	(- 45.55 (+ 44.05
6	+ 22.8	+10.8	+15.0	-4.0	+ 9.05	-.398	.398	6.4	-21.9	(- 3.49 (+44.59

Table 44. Elastic Theory--Steel Required--Bents 3 and 4

Pt.	M	N	e	t	d	E	Mult $\frac{f'cbd^2}{3}$	NE	c	Top As $\frac{As-N(e-c)}{fyc'}$	Top Bars	Bot.A's $\frac{NE-Mult}{fyd'}$	Bot Bars
1	-48.87	47.55	12.35	11.5	9.5	1.34	54.2	63.8	7.04*	3.48	3 #10	.767	1 #10
	+ 8.33	21.49	4.65	11.5	9.5	.693	54.2	14.9	6.95	--	--	(197)	1 #10
2	-146.82	44.21	39.9	16.5	14.5	3.84	126.	170.	11.0	7.05	3#11,2#10	2.11	2 #10
3	-245.08	41.61	70.6	21.5	19.5	6.6	228.	274.	14.73	9.13	6 #11	1.575	2 #10
4	-185.15	24.81	89.5	21.0	19.0	8.16	216.4	202.5	13.91	7.48	5 #11	--	--
5	- 45.55	16.93	32.2	15.	13.0	3.14	101.3	53.4	9.52	2.5	2 #11	--	--
	+ 44.05	6.90	76.5	15.	13.0	6.83	101.3	47.1	9.52	--	--	(2.62)	2 #11
6	- 3.49	13.81	3.03	9.	7.	.46	29.4	6.36	5.11	.552	1 #4	--	--
	+ 44.59	3.17	169.	9.	7.	14.3	29.4	44.7	5.05	(1.835)	1#11,1#10	(5.2)	2 #11, 2 #10

* Place bars in two rows.

Table 45. Bond and Shear--Bents 3 and 4

Pt.	V max.	C	bc	V/bc = v	o	oc	V/ oc = u	Stirrups
1	9.015	7.0	126	71.5	14.5	101.5	88.8	#2 hoops @24"c/c
2	9.015	10.9	196	46.0	28.5	311	29.0	do
3	9.015	14.5	261	34.5	33.5	486	18.5	do
4	31.91	13.9	250	123.5	23.5	327	97.5	See below
5	23.44	9.5	171	137.0	19.0	180.5	130	do
6	15.11	5.05	90.9	166.0	28.5	144	105	do

Table 46. Maximum Moments - Bents 2 and 5

Pt.	Dead Load	Live Load Full	Live Load Left	Live Load Right	Live Load Center	Temperature +60°	Temperature -60°	Wind Left	Wind Right	Max. Comb.
1	- 34.85	-15.25	- 7.6	- 7.6	-11.23	-.042	+.042	+40.93	- 2.58	(- 52.68 (+ 6.12
2	-101.6	-45.7	-21.75	-21.75	-33.7	-.141	+.141	+81.48	- 7.72	-155.12
3	-169.5	-76.4	-37.9	-37.9	-56.1	-.234	+.234	+85.37	-12.88	-258.98
4	-121.15	-57.2	-17.	-16.4	-48.7	-.305	+.305	+74.3	-16.7	-195.35
5	- 7.4	- 6.24	+15.25	-20.4	-11.6	-.351	+.351	+36.4	-19.3	(- 47.45 (+ 44.6
6	+ 23.8	+11.16	+16.1	- 4.29	+ 9.7	-.398	+.398	+ 6.4	-21.9	(- 4.4 (+ 46.69

In a similar manner, the moments induced in the bent under the four conditions of live loading are computed. The effect of temperature change and wind loading will produce the same effect as on the other interior bents -- 3 and 4.

The combined moments and their maximum effect on the bent are shown in Table 46. A tabulation of the value of shear and thrust at the points on the bent is contained in Table 47. The area of steel required at the various points on the bent and the resulting values of bond and shear are shown in Tables 48 and 49.

Exterior bents - 1 and 6

In the previous calculations for moments induced by wind loading it was assumed that the two exterior bents would have the same shape as the interior bents but would be only half as wide. The assumption of the reduced width was based on the requirements set forth in Section 701(c) of the A. C. I. Building Code. This section states that, for this design, the shear at all supports except the first interior may be taken as WL/2. Since the load transferred to the two exterior bents, is only one-half of that at bents 3 and 4 it is both conventional and economical to reduce their size.

The calculations for final moments are similar to those illustrated previously and only the final results will be shown here. The results are contained in Tables 50, 51, 52 and 53.

Table 47. Thrusts and Shears - Bents 2 and 5

Point:	1		2		3		4		5		6	
Condition:	N	V	N	V	N	V	N	V	N	V	N	V
Dead Load	31.89	6.77	28.79	6.77	26.19	6.77	15.62	19.49	11.05	12.39	8.69	8.1
L.L. Full	11.2	3.05	11.2	3.05	11.2	3.05	7.34	9.6	5.25	5.25	3.35	2.7
L.L. Left	8.38	1.51.	8.38	1.51	8.38	1.51	4.57	6.4	3.78	3.17	.84	1.5
L.L. Right	2.8	1.51	2.8	1.51	2.8	1.51	2.71	3.17	2.71	3.17	2.71	3.2
L.L. Center	5.58	2.23	5.58	2.23	5.58	2.23	4.72	5.95	4.72	5.95	2.86	2.7
Wind Left	-7.81	-15.	-7.81	-11.	-7.81	-7.	-8.38	-11.25	-6.68	-9.25	-4.55	-7.8
Wind Right	7.81	.55	7.81	.55	7.81	.55	4.35	6.97	4.35	6.97	4.35	6.9

Table 48. Plastic Theory--Bents 2 and 5

Pt.	M	N	e	t	d	E	Mult.	NE	C	Top As	Top Bars	Bot. A's	Bot. Bars
1	-52.68	50.90	12.4	11.5	9,5	1.35	54.2	68.8	7.05	3.28	3.#11	1.17	1.#10
	+ 6.12	24.08	3.05	11.5	9.5	.566	54.2	13.65	6.95	--	--	(1.19)	1 #10
2	-155.12	47.80	39.0	16.5	14.5	3.77	126.	180.	11.10	7.35	5 #11	2.6	1#11, 1#10
3	-258.98	45.20	68.4	21.5	19.5	6.42	228.	290.	14.75	9.55	7 #11	1.44	1 #11
4	-195.35	27.31	85.6	21.	19.0	7.84	216.4	214.	13.91	7.85	5 #11	--	--
5	- 47.45	18.57	30.6	15.	13.0	3.0	101.3	55.7	9.5	2.61	2 #11	--	--
	+ 44.6	8.15	65.5	15.	13.0	5.92	101.3	48.3	9.5	--	--	(2.64)	2 #11
6	- 4.4	15.69	3.36	9.	7.0	.49	29.4	7.68	5.11	1.18	1 #10	--	--
	+ 46.89	4.918	115.	9.	7.0	9.8	29.4	47.8	5.11	2.21	2 #10	(5.36)	2 #11 2 #10

Table 49. Bond and Shear--Bents 2 and 5

Pt.	V max.	C	bc	V/bc = v	o	oc	V/ oc = u	Stirrups
1	9.55	7.0	126.	70.1	14.5	101.5	94.	#2 Hoops 24" c/c
2	9.55	11.0	198.	48.2	34.	375.	25.4	do
3	9.55	14.65	264.	35.2	35.	512.	18.7	do
4	34.73	13.9	250.	138.5	30.	417.	83.5	See below
5	25.31	9.5	171.	148.	20.	189.	134.	do
6	18.24	5.11	92.	198.5	28.	143.	127.5	do

For stirrups use #3 Hoops:

$$\text{max. } 1/s = \frac{130 \times 18}{4400} = .53$$

$$\text{min. } 1/s = \frac{48.5 \times 18}{4400} = .194$$

$$N = 6 \times 25\ (.724) = 108.5$$

$$\text{Index} = 37.5/.336 = 112.$$

Use 52 @ 2"
32 @ 3"
18 @ 4"
8 @ 6"

Table 50. Maximum Moments - Bents 1 and 6

Pt.	Dead Load	Live Load Full	Live Load Left	Live Load Right	Live Load Center	Temperature +60°	Temperature -60°	Wind Left	Wind Right	Max. Comb.
1	-16.05	- 7.2	- 3.53	- 3.53	- 5.23	-.084	+.084	20.47	- 1.78	(- 24.54 (+ 4.51
2	-48.25	-21.3	-10.1	-10.1	-15.65	-.282	+.282	40.74	- 3.86	- 73.41
3	-80.5	-35.5	-17.67	-17.65	-26.1	-.468	+.468	42.69	- 6.44	-122.44
4	-57.48	-26.6	- 7.92	-16.95	-22.65	-.61	+.61	37.15	-.8.35	- 92.43
5	- 3.45	- 2.9	+ 7.1	- 9.5	- 5.4	-.702	+.702	18.2	- 9.65	(- 23.30 (+ 22.55
6	+11.4	+ 5.4	+ 7.5	- 2.0	+ 4.53	-.796	+.796	3.2	-10.95	(+ 22.9 (- 3.34

Table 51. Thrusts and Shears - Bents 1 and 6

Point:	1		2		3		4		5		6	
Condition:	N	V	N	V	N	V	N	V	N	V	N	V
Dead Load	14.7	3.21	13.0	3.21	11.7	3.21	6.83	8.62	5.03	5.47	3.47	2.80
L.L. Full	5.2	1.42	5.2	1.42	5.29	1.42	3.39	4.46	2.42	2.77	1.55	1.28
L.L. Left	3.4	.7	3.4	.7	3.4	.7	2.13	2.98	1.76	1.47	1.39	.023
L.L. Right	1.3	.7	1.3	.7	1.3	.7	1.26	1.48	1.26	1.48	1.26	1.48
L.L. Center	2.6	1.04	2.6	1.04	2.6	1.04	2.2	2.77	2.2	2.77	1.33	1.27
Wind Left	-3.9	-7.5	-3.9	-5.5	-3.9	-3.5	-4.19	-5.6	-3.34	-4.6	-2.28	-3.88
Wind Right	+3.9	.257	3.9	.257	3.9	.257	2.17	3.48	2.17	3.48	2.17	3.48

Table 52. Plastic Theory--Steel Required--Bents 1 and 6

Pt.	M	N	e	t	d	E	Mult.	NE	C	Top As	Top Bars	Bot. A's	Bot. Bars
1	-24.54	23.8	12.35	11.5	9.5	1.34	27.1	31.9	7.04	1.535	1 #11	.384	1 #10
	+4.51	10.8	4.65	11.5	9.5	.70	27.1	7.5	6.95	--	--	(.14)	1 #10
2	-73.41	22.1	39.9	16.5	14.5	3.84	63.0	85.0	11.0	3.54	2#11,1#10	1.06	1 #10
3.	-122.44	20.8	70.6	21.5	19.5	6.6	114.0	137.	14.7	4,56	3# 11	.79	1# 10
4	-92.43	12.39	89.5	21.0	19.0	8.16	108.2	101.	13.91	3.74	2#11,1#10	--	--
5	-23.3	8.46	31.7	15.	13.0	3.1	50.65	26.2	9.52	1.23	1 #10	--	--
	+2.34	3.45	78.5	15.	13.0	7.00	50.65	24.1	9.52	--	--	1.35	1#11
6	+22.9	6.90	4.06	9.	7.0	.548	14.7	3.78	5.11	.10	--	--	--
	+22.9	1.58	162.0	9.	7.0	13.69	14.7	21.6	5.07	(.807)	1 #10	(2.48)	1#11, 1#10

Table 53. Shear and Bond--Bents 1 and 6

Pt.	V. max	C	bc	V/bc = v	o	oc	V/ oc = u	Stirrups
1	4.51	6.77	61.	74.	14.5	98.	46.	#2 Hoops, 24" c/c
2	4.51	10.9	97.	46.5	23.5	256.	175.	do
3	4.51	14.6	130.	34.7	19.0	277.	163.	do
4	15.95	13.9	123.5	129.	14.5	202.	79.	See below
5	11.77	9.52	84.6	139.	9.5	90.4	130.	do
6	7.76	5.11	46.	169.	14.	71.5	180.5	do

For stirrups use #2 Hoops:

$$\text{Max. } 1/s = \frac{95 \times 9}{2000} = .428$$

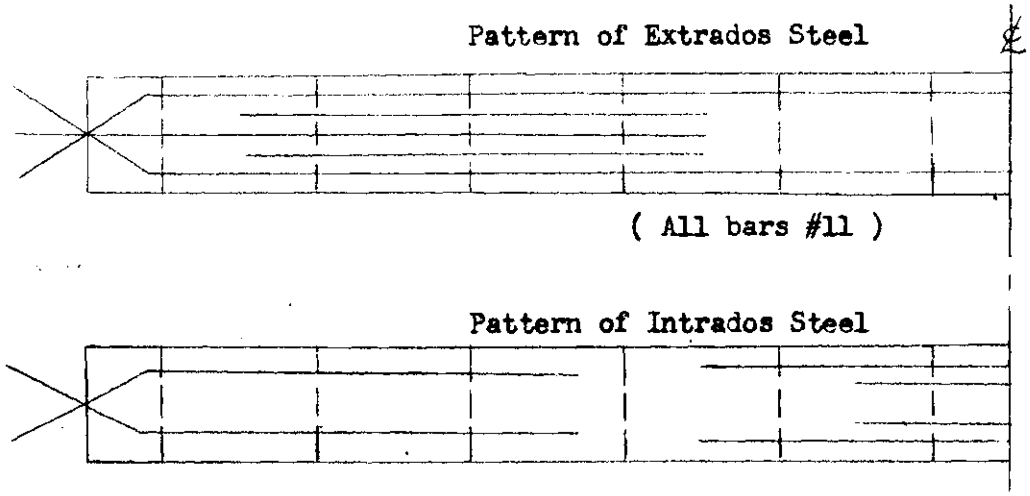
$$\text{Min. } 1/s = \frac{39 \times 9}{2000} = .176$$

$$\overline{.604}$$

$N = 150\ (.604) = 91.$

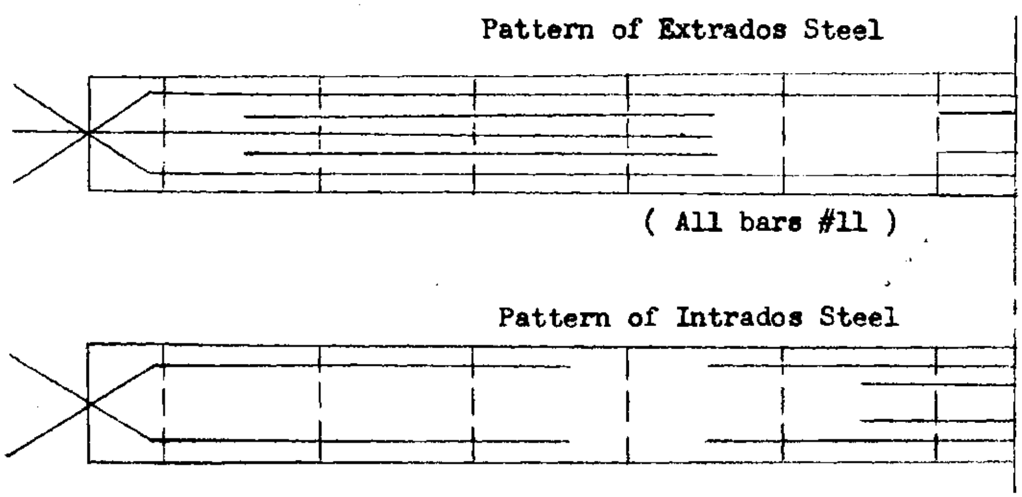
Index $= 37.5/_{.252} = 149.$

Use: 12 @ 2"
44 @ 3"
24 @ 4"
12 @ 6"



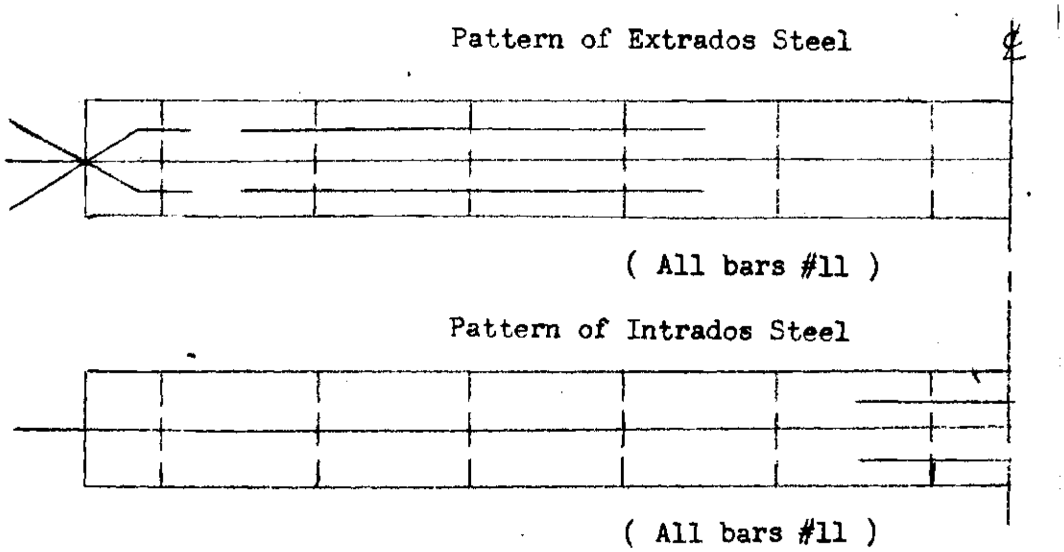
Steel in Bents 3 and 4

Figure 28



Steel in Bents 2 and 5

Figure 29



Steel in Bents 1 and 6

Figure 30

DESIGN CALCULATIONS--RIGID FRAME--ULTIMATE THEORY

To determine the section depths the moments computed for the three-hinged constant-section bent of the Conventional Theory are again used. As in the Plastic Theory proposed by Mr. Whitney, a safety factor of 2.5 will be used. However, the method of application will be different from that followed in the Plastic Theory. Rather than take 0.4 of the ultimate concrete and steel stresses, the moments obtained by analysis will be multiplied by 2.5, and the resulting values used to obtain the required section depths and steel areas. This procedure was adopted merely for convenience as the Portland Cement Association has published a pamphlet on Ultimate Design which contains tabular data similar to that available for conventional design methods.

Choosing the ultimate stress values as 3,000 psi. for concrete and 50,000 psi. for steel the K value for balanced design (from the pamphlet cited above) is 1,084. From the relationship, $d = (M/Kb)^{\frac{1}{2}}$, the required section depths are determined as shown in Table 54.

Although the depths determined by this analysis are approximately 4% less than those required by the Plastic Theory analysis, the same critical bent dimensions will be used as in that case. This is, $d_A = 10$ in., $d_B = 24$ in. and $d_c = 6$ in.

Therefore, for all bents the maximum combined design moments will be 2.5 times greater than those computed for the Plastic Theory analysis. Since, by the method of safety factor application, we are designing for a moment 2.5 times greater than is actually present, it is necessary to increase the values of thrust obtained by the previous analysis by the same factor.

The method of obtaining the required steel areas is very similar to that followed in the Conventional Design Theory. Knowing the value of the thrust, N, and the distance of its eccentricity about the tensile steel, E, the value NE is computed. Next, the values K and F, dependent on the qualities of material and section dimensions are found. Should the value KF exceed NE, no compression steel is required and the tensile steel area is determined from the formula

$$A_s = 1000N(e/jd - 1)/f_s$$

If NE exceeds KF, compression steel is required and the procedure is as follows:

a. Determine the compressive steel stress using the relationship,

$$f'_s = (k_o d - d')f_s/d - k_o d \quad \text{(But not greater than } f_s\text{)}$$

b. Using the f'_s value obtained above compute the area of compression steel, which is

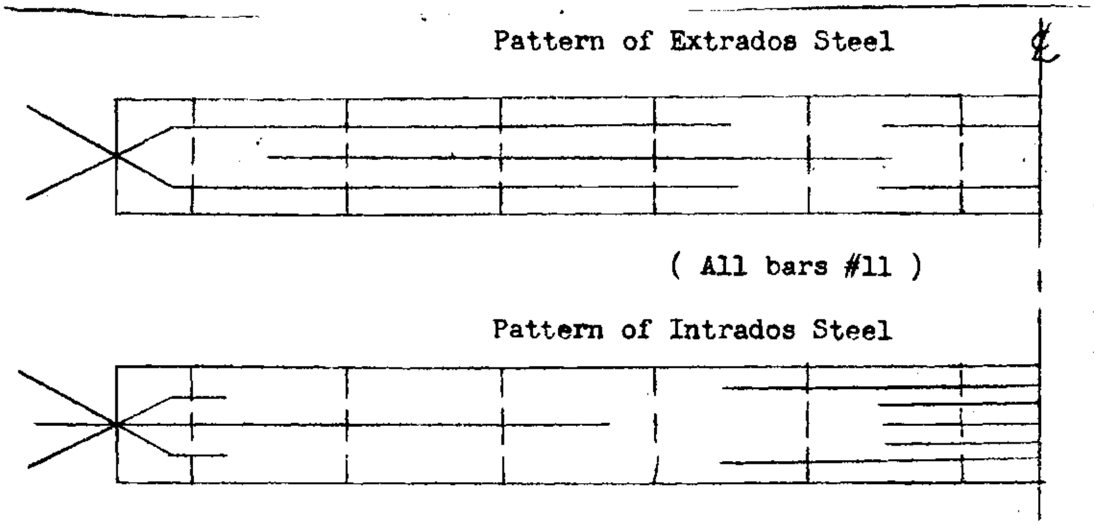
$$A'_s = 12{,}000(NE - K_oF)/f'_s(d - d')$$

c. Solve for the moment arm "jd" where

$$jd = NE/(K_oF/j_od + NE - K_oF/d - d')$$

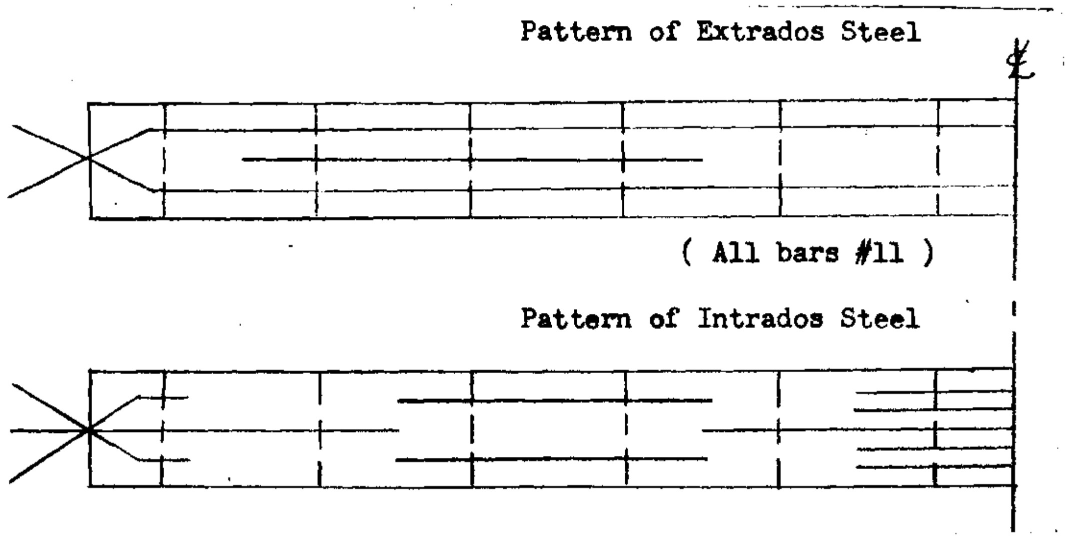
d. Compute the tensile steel area,

$$A_s = 1000N(e/jd - 1)/f_s$$



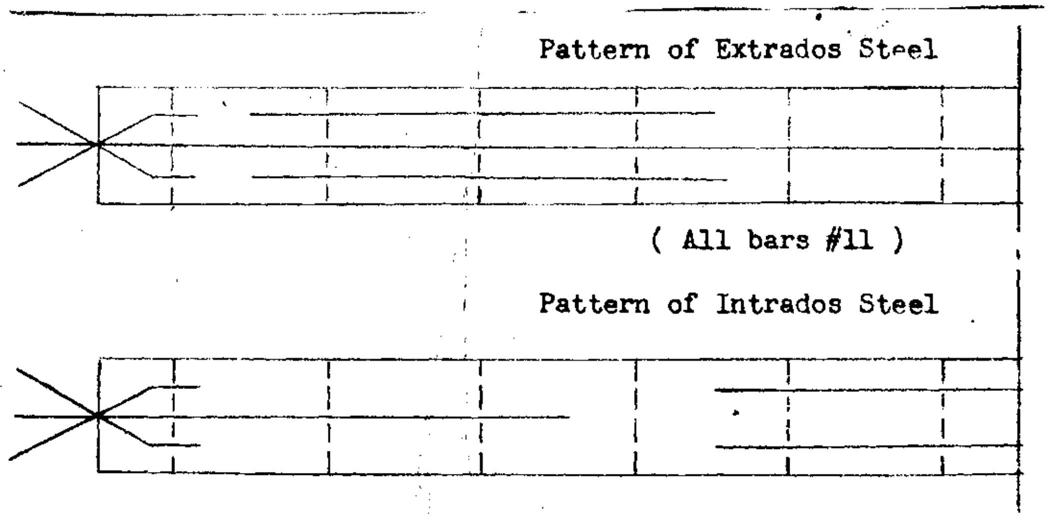
Steel in Bents 3 and 4

Figure 31



Steel in Bents 2 and 5

Figure 32



Steel in Bents 1 and 6

Figure 33

Table 54. Determination of Section Depths.

Point	M(ft.-kips)	M(in.-lbs.)	2.5M	2.5M/Kb	$(2.5M/Kb)^{\frac{1}{2}}$
1	- 55.75	669,000	1,673,000	85.8	9.26"
2	-167.5	2,010,000	5,030,000	258.	16.06
3	-278.	3,335,000	8,340,000	427.	20.64
4	-212.75	2,555,000	6,380,000	327.5	18.1
5	- 45.5	546,000	1,364,000	70.0	8.36
6	+ 6.0	72,000	180,000	9.23	3.04

Table 55. Ultimate Theory--Steel Required--Bents 3 and 4

Pt.	M Max. l-k	N k.	e = M/N	t	d	e'	E	F	NE	K=NE/F	jd	e/yd - 1	Top as	Bars Top & Bot.	f's	Bot. A's
1	-122.	119.	12.35	11.5	9.5	16.10	1.34	.136	160.0	1175.	7.25	1.22	2.91	2#11 1#10	44,200	.458
2	+ 20.4	53.8	4.65	11.5	9.5	8.30	.693	.136	37.3	274.	8.65	(0)	--	--	--	(0)
	367.	110.5	39.9	16.5	14.5	46.15	3.84	.316	425.	1350.	11.3	3.08	6.78	(2#11 (3#10 1#11	50,000	1.58
3	-612.	104.2	70.6	21.5	19.5	79.35	6.6	.566	688.	1220.	15.3	4.25	8.85	6#11 1#10	50,000	1.0
4	-464.	62.1	89.5	21.0	19.0	98.0	8.16	.541	506.	935.	15.15	5.48	6.34	2#11 3#6	--	--
5	-114.	42.4	32.2	15.	13.0	37.7	3.14	.254	133.5	525.	13.2	1.85	1.57	1#11	--	--
	-10.	17.3	76.5	15.	13.0	82.0	6.83	.254	118.	464.	11.5	6.14	--	1#11 1#10	--	(2.12)
6	- 8.73	34.6	3.03	9.	7.0	5.53	.46	.074	15.9	214.	6.45	0	0	--	--	--
	+111.	7.94	169.	9.	7.0	171.5	14.3	.074	112.	1520.	5.2	32.	(2.11)	(1#11 (1#10 2#11) 3#10)	35,600	(5.05)

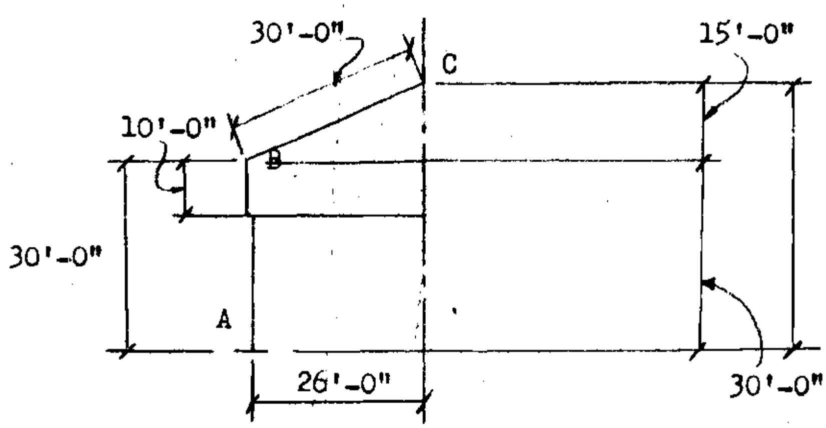
Table 56. Ultimate Theory--Steel Required--Bents 2 and 5

Pt	M Max.	N	e	t	d'	e'	E	F	NE	$K = \frac{NE}{F}$	jd	e/jd-1	Top As	Bars Top & Bot.	Bot. A's
1	-131.5	12.7	12.4	11.5	9.5	16.2	1.35	.136	171.	1260.	7.25	1.23	3.12	2#11 1#10	.87
	+15.38	60.2	3.05	11.5	9.5	6.8	.566	.136	34.1	251.	8.67	--	7.15	1#10	(.97)
2	-397.	119.4	39.0	16.5	14.5	45.1	3.77	.316	450.	1430.	11.3	2.99	7.15	3#11,2#10 1#11,1#10	2.05
3	-647.	113.0	68.4	21.5	19.5	77.	6.42	.566	725.	1130	15.15	4.09	9.38	6#11 1#11	1.51
4	-488.	68.2	85.6	21.	19.0	94.	7.84	.541	535.	990.	14.9	5.32	7.24	5#11	--
5	-118.2	46.4	30.6	15.	13.0	36.1	3.0	.254	139.	547.	11.4	2.17	2.01	1#11,1#10	--
	-111.5	20.35	65.5	15.	13.0	71.	5.92	.254	120.5	475.	11.5	5.16	--	1#11, 1#10	(2.1)
6	- 11.0	39.2	3.36	9.	7.0	586.	.49	.074	19.2	260.	6.38	--	.072	1#10	--
	+117.0	12.3	115.	9.	117.4	9.8	.074	.074	120.	1625.	5.22	21.4	(2.72)	(1#11 (1#10 1#11) 3#10)	(5.28)

Table 57. Ultimate Theory--Steel Required--Bents 1 and 6

Pt.	M Max.	N	e	t	d	e'	E	F	NE	$K = \frac{NE}{F}$	jd	e/yd-1	Top A_s	Bars Top & Bot.	Bot A's
1	-61.2	59.5	12.35	11.5	9.5	16.15	1.34	.068	78.7	1160.	7.2	1.24	1.48	1#11 1#10	.17
2	+11.28	27.	4.65	11.5	9.5	8.40	.70	.068	18.9	278.	7.2	.165	--	-- 1#10	(.107)
3	-183.5	55.3	39.9	16.5	14.5	46.15	3.84	.158	212.	1340.	11.25	3.1	3.42	(2#11, (1#10. 1#10	.78
	-307.	52.	70.6	21.5	19.5	79.35	6.6	.285	343.	1205.	15.0	4.28	4.55	3#11 1#10	.465
4	-231.	31.	89.5	21.0	19.0	98.	8.16	.271	253.	935.	15.15	5.46	3.39	3#11	—
5	-58.4	21.2	31.7	15.	13.0	37.2	3.1	.126	65.7	522.	11.42	2.26	.955	1#10	—
	+56.3	8.64	78.5	15.	13.0	84.	7.00	.126	60.5	480.	11.5	6.3	—	—	(1.09)
6	- 5.85	17.25	4.06	9.	7.0	5.56	.548	.037	9.45	255.	6.4	.025	.086	1#10	—
	+ 5.62	3.95	162.0	9.	7.0	164.5	13.69	.037	54.0	1460.	5.65	29.	.945	2#10	(2.29)

DESIGN CALCULATIONS--HIPPED PLATE STRUCTURE



Live Load = 20 psf.
Wind Load = 20 psf.

On inclined member, BC,

L.L. = 20 x 26/30 = 17.35 plf.
W.L. = 20 x 15/30 = 10.0 plf.

Dimensions of Structure

Figure 34

The dimensions of the hipped plate structure are as shown in Figure 34. It is a simply supported structure and is composed of two 50'-0" long spans. As in previous calculations the live load and wind load are assumed to be 20 psf. The conditions of loading will be

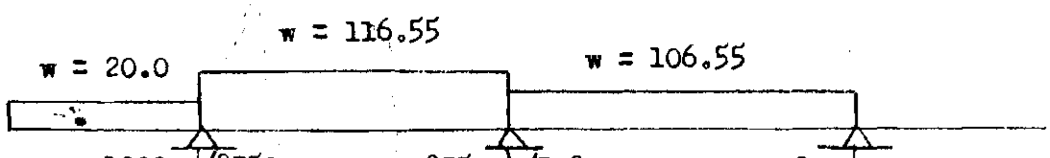
a. Dead Load

b. Live Load full

c. Live Load one-half

d. Wind Load

The structure will be designed using the A.C.I. Building Code specifications, with allowable stresses of 3,000 psi., and 20,000 psi., for concrete and steel respectively. A total slab thickness of 8.25 inches is assumed giving a projected dead weight of 89.2 plf. The slab is analyzed by moment distribution; the procedure is shown in the following illustrations.



-1000.	+8750.	-8750.	+7980.	-7980.	0
	-7750.	+ 385.	+ 385.	+7980.	
	+ 192.5	-3875.	+3990.	+ 192.5	
	- 192.5	- 62.	- 62.	- 192.5	
	- 31.3	- 96.	- 96.	- 31.3	
	+ 31.3	+ 96.	+ 96.	+ 31.3	
-1000.	+1000.	-12302.	+12302.	0.	0.

$d^2 = M/Kb = 52.1$, $d = 7.25"$ with $h = 8.25"$

Slab Analysis--Case I--D.L. + L.L. Full + Wind

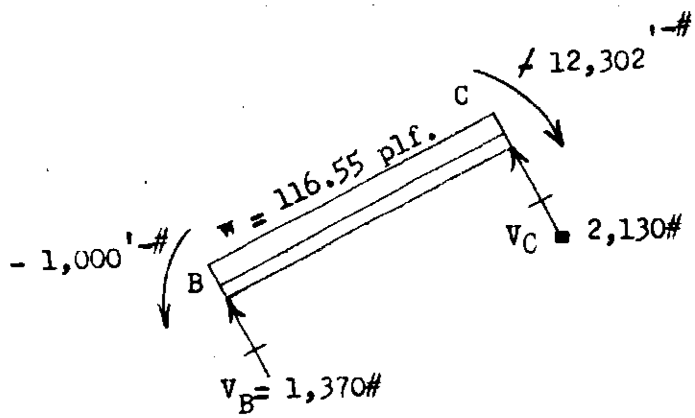
Figure 35

-1000.	+ 8750.	-8750.	+6690.	-6690.	0.
	- 7750.	+ 530.	+ 530.	+6690.	
	+ 265.	-3875.	+3345.	+ 265.	
	- 265.	+ 265.	+ 265.	- 265.	
	+ 132.5	- 132.5	- 132.5	+ 132.5	
	- 132.5	+ 132.5	+ 132.5	- 132.5	
-1000.	+ 1000.	-11830.	+11830.	0.	0.

Slab Analysis--Case II--D.L. + L.L. 1/2 + Wind

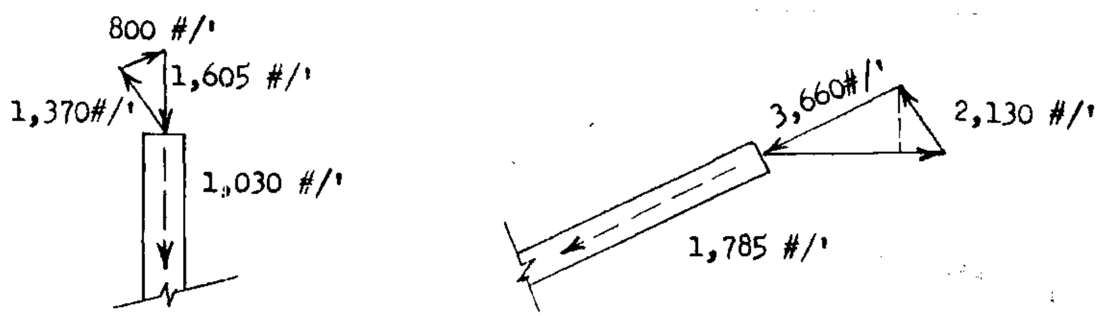
Figure 36

The loading condition analyzed in Figure 35 requires the thicker slab and will govern the design. The forces acting on the plates will now be computed. They will be determined by the vertical reactions induced by the slab loading and are, in turn, resolved into components in the plane of the plates. For convenience, the plate action in AB, Figure 34, will be designated as Plate 2.



Reactions Due to Slab Loading - Plate 2

Figure 37



(a) Plate 1 (-2635#/-') (b) Plate 2 (-4645#/-')

Forces Acting in Planes of Plates

Figure 38

The plates are considered to act as simple beams; therefore, the moments induced at the center line of each plate may be determined by the equation,

$$M_o = \frac{WL^2}{8}$$

The value M_o, is termed the "central bending moment". Applying the above equation to the two plates it is found that the central bending moments for Plates 1 and 2 are 824.0 ft.-kips and 1,450 ft.-kips respectively.

The "fixed end shears" used in the moment distribution analogy for determining the longitudinal edge force, N, are

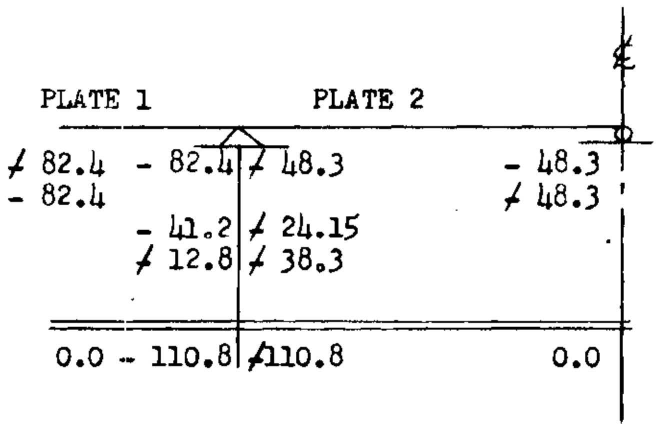
$$\frac{M_o}{h.} = \frac{824}{10} = 82.4 \text{ kips for Plate 1}$$

and

$$\frac{M_o}{h_2} = \frac{1450}{30} = 48.33 \text{ kips for Plate 2}$$

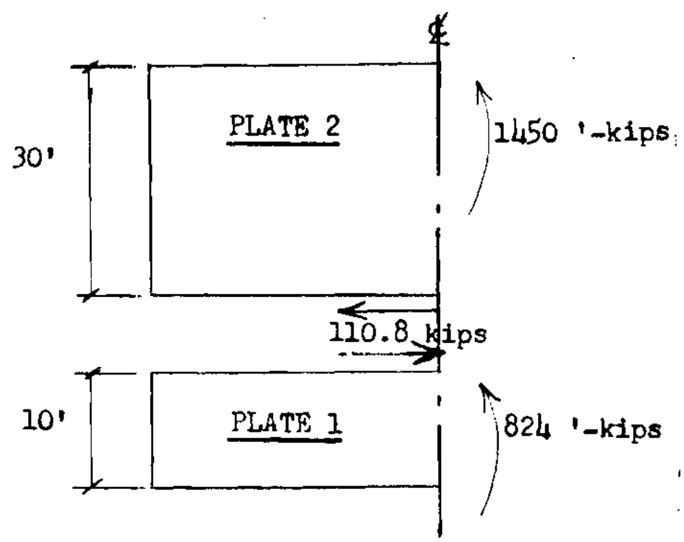
The corresponding stiffness factors, K, are 10.0 for Plate 1 and 30.0 for Plate 2. The application of the moment distribution analogy is illustrated in Figure 39.

Knowing both the amount and direction of the value, N, the resulting plate moments are determined as shown in Figure 40, to be $\neq$ 210 ft.-kips for Plate 2 and - 270 ft.-kips for Plate 1.



Determination of Longitudinal Force, N.

Figure 39



Total Bending Moments in Plates

Figure 40

Using the expression

$$f = \frac{N}{A} \pm \frac{6M}{bh^2}$$

the extreme fibre stresses at the mid-points of the plates at the top and bottom fibres will be:

For Plate 2: $\dfrac{110.8}{2970} \pm \dfrac{72 \times 210}{8.25 \times (360)^2}$

$$= (0.0372 \pm 0.0141) \text{ ksi.} = + 23.1 \text{ psi., C -(Top)}$$

$$= + 51.4 \text{ psi., C -(Bottom)}$$

For Plate 1: $\dfrac{-110.8}{990} \pm \dfrac{72 \times -270}{8.25 \times (120)^2}$

$$= (-0.112 \pm 0.163) \text{ ksi.} = + 51.4 \text{ psi., C -(Top)}$$

$$= - 273.0 \text{ psi., T -(Bottom)}$$

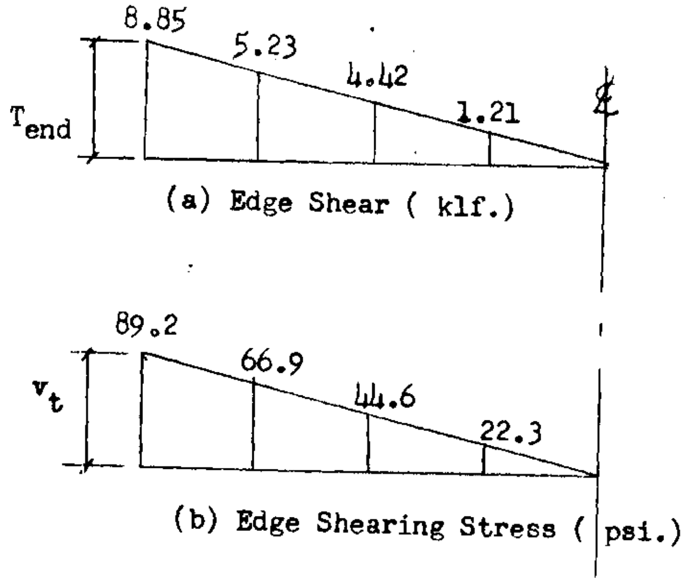
Since the value of N must be equal to the area of the shear diagram at the plate edge (Eq.18a), the value of the shear at the end of the plates is:

$$T = \frac{4N}{L} = \frac{4 \times 110.8}{50} = 8.85 \text{ kips}$$

The resulting shearing stress, v_t, will be

$$v_t = 8{,}850/8.25 \times 12 = 89.2 \text{ psi.}$$

These conditions are illustrated in Figure 41.

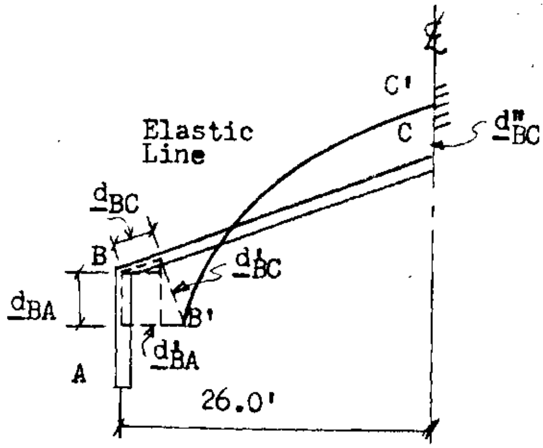


Shear at Plate Edges

Figure 41

The effects of the plate secondary bending moments will now be considered. From a consideration of the loadings acting on the plates (Figure 38) the deflections of the plates in planes perpendicular to those in which the loads act can be obtained. The resulting internal moments produced by the loadings (Figure 38) are shown in Figure 40 as 210 ft.-kips clockwise for Plate 2 and 270 ft.-kips counter-clockwise for Plate 1. Due to the inclination of Plate 2 the distortions of the structure will occur in two planes. The maximum deflection of the plates under the applied loadings will occur at the midpoints of the plates (as in a simple beam subjected to uniform loading) and the deflection curve of the structure will be obtained at the plates midpoint from a consideration of these deflections.

Figure 42 illustrates the distortion of the structure. Point B tends to move to a new location, B'. This gives the plate, BC, the effect of a beam fixed at one end and guided at the other.



Deflection in Plates at Vertical Axis Through Mispoint of Span

Figure 42

The deflections of the structure are obtained as follows from Figure 42:

$$d_{BC} = \frac{5WL^4}{384EI} = \frac{5ML^2}{4Ebh^3} = \frac{5 \times 210 \times 12 \times 2500 \times 144}{4 \times E \times 8.25 \times 30^3 \times 12^3} = \frac{2.95}{E} \text{ in.}$$

$$d_{BA} = \frac{5ML^2}{4Ebh^3} = \frac{5 \times 270 \times 12 \times 2500 \times 144}{4 \times E \times 8.25 \times 10^3 \times 12^3} = \frac{102.6}{E} \text{ in.}$$

$$d'_{BC} \times 26/30 = d_{BC} \times 15/30 + d_{BA}$$

$$d'_{BC} = \frac{15}{26} \times \frac{2.95}{E} + \frac{102.6}{E} \times \frac{30}{26} = \frac{119.9}{E} \text{ in.}$$

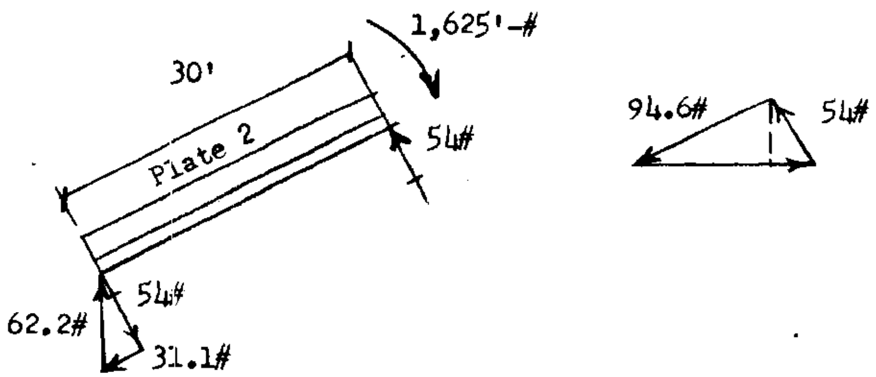
$$d''_{BC} = d_{BC} \times \frac{26}{15} = \frac{5.12}{E} \text{ in.}$$

Total plate deflection $= \underline{d}'_{BC} + \underline{d}''_{BC} = \frac{125}{E}$ in.

The moment at C will be $\frac{3EI\underline{d}}{L^2}$

$$M = \frac{3E \times 12 \times (8.25)^3}{12 \times (30)^2 \times (12)^2} \times \frac{125}{E} = 1.625 \text{ ft.-kips}$$

The vertical reactions produced by this secondary bending moment are found and resolved to determine their loading effect on the plates. These vertical reactions actually occur at the midpoint of the plates where the deflection under loading is the maximum. Consequently, the plate loadings determined by resolution will not be constant along the plate edges but will vary parabolically, being a maximum at the center. The procedure is illustrated in Figure 41.



Forces in Plates

Figure 43

From the above Figure, it can be seen that the maximum center loading on Plate 1 is 62.2 lbs., acting upward and on Plate 2, 125.7 lbs., acting downward. With parabolic loading, the new central bending moments are 16.57 ft.-kips for Plate 1 and 32.7 ft.-kips for Plate 2. Applying once more the "fixed-end shear" analogy illustrated in Figure 44, the edge force, N, is found to be 1.46 kips, opposite in sign to the value of 110.8 kips calculated previously. Therefore, the effect of the secondary bending moments will be to decrease the values of bending moments and stresses obtained before. Hence, it will be safe to use the values previously determined in the subsequent design.

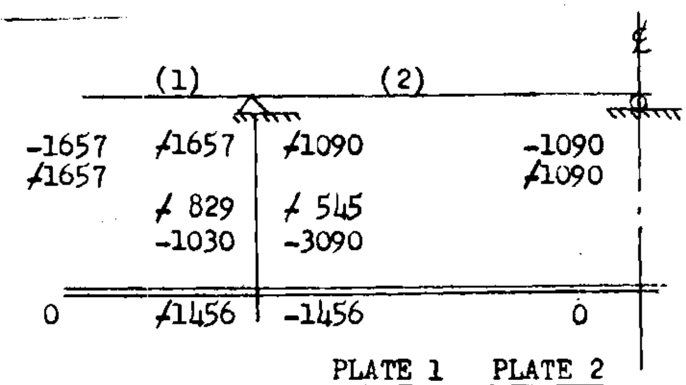
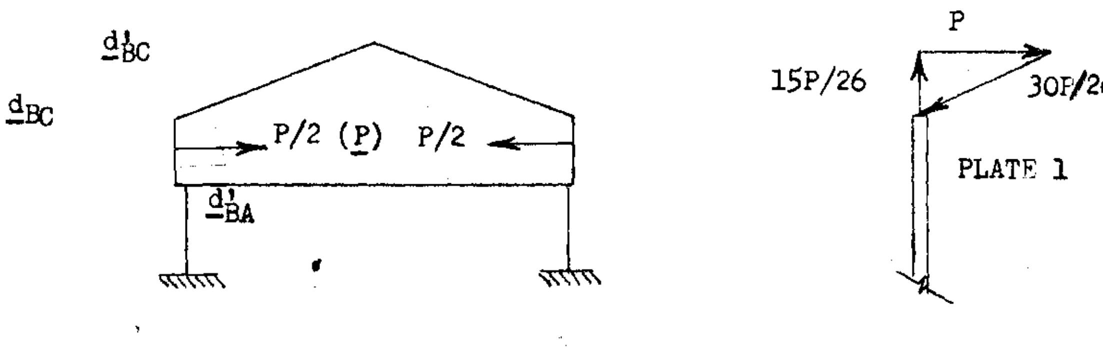


	PLATE 1	PLATE 2
Previous shear	-110.8	+110.8
Plate shear	+ 1.45	- 1.45
New shear	-109.35	+109.35

Determination of Longitudinal Force, N.

Figure 44



(a) (b)

Forces in Tie Beam (Diaphragm)

Figure 45

The deflection $\underline{d}'_{BA}$ (Figure 42) will be the same as that produced by some concentrated load, P, acting at the midpoint of the plates. This loading will produce a reaction equal to P/2 at each junction of the plate to the tie beam or end diaphragm. Since the structure is symmetrical the total force in the end diaphragm will be the equivalent of two reactions and will be equal in magnitude to P. The value of P is obtained by determining the resulting deflections at the midpoints of the plates and equating them to the plate deformations shown in Figure 42.

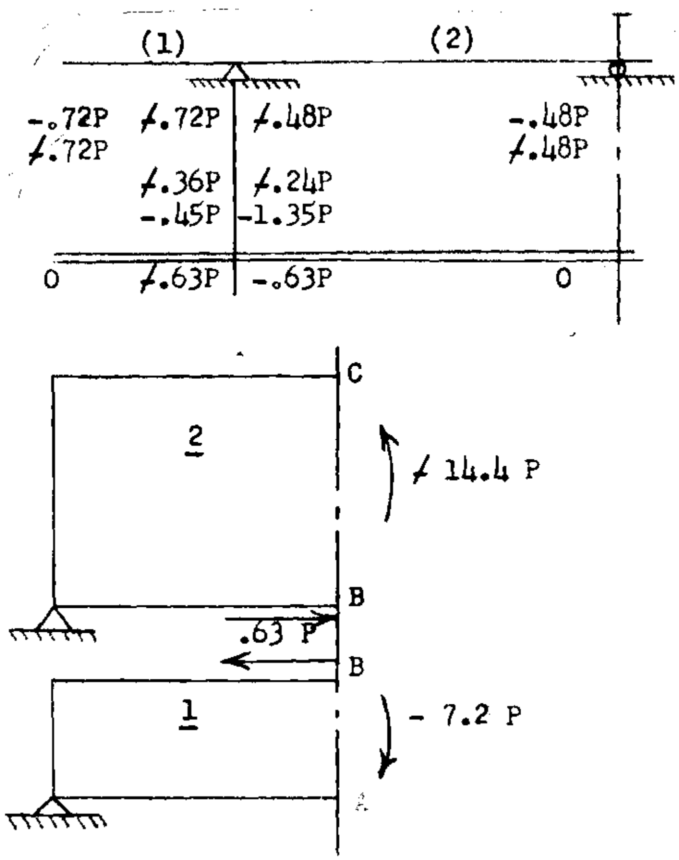
Figure 45 shows the forces, P, acting in the tie beam and the resulting loads taken by the plates. For a concentrated load, the moment induced in Plate 1 will be

$$M_{o1} = 15P/26 \times 50/4 = 7.2 \text{ P ft.-kips}$$

and in Plate 2,

$$M_{o2} = 30P/26 \times 50/4 = 14.4 \text{ ft.-kips}$$

Applying the moment distribution analogy, the total edge force is found to be 0.63 P kips. The resulting plate moments (determined by the same method shown in Figure 40) are 23.85 P ft.-kips counter-clockwise for Plate 2 and 4.05 P ft.-kips clockwise for Plate 1.



Determination of Longitudinal Force, N, and Resulting Moments in Plates

Figure 46

In simple beam action, the deflection, d, produced by a concentrated load at the midpoint of the plates is:

$$\underline{d} = ML^2/12EI \text{ where } M = \frac{PL}{4}$$

Figure 45 shows the deflections, $\underline{d}_{BC}$, $\underline{d}'_{BC}$ and $\underline{d}'_{BA}$ produced in the structure by the force P acting in the tie beam. These deflections correspond to those induced by the plate deflections shown in Figure 42. The deflections in terms of P are:

$$\underline{d}_{BC} = \frac{23.85P \times 12 \times (50 \times 12)^2 \times 12}{12E \times 8.25 \times (30 \times 12)^3} = \frac{0.268P}{E} \text{ in.}$$

$$\underline{d}_{BA} = \frac{4.05P \times 12 \times (50 \times 12)^2 \times 12}{12E \times 8.25 \times (10 \times 12)^3} = \frac{1.23P}{E} \text{ in.}$$

$$\underline{d}'_{BC} = \frac{15}{26}\,\underline{d}_{BC} + \frac{30}{26}\,\underline{d}_{BA}$$

$$\underline{d}'_{BA} = \frac{26}{30}\,\underline{d}_{BC} + \frac{15}{30}\,\underline{d}'_{BC} \qquad = \frac{1.019P}{E} \text{ in.}$$

Since the distortions produced by the force, P, must be the same as those from the plate deflections

$$\underline{d}'_{BA} = \underline{d}'_{BA}$$

$$\frac{1.02P}{E} = \frac{62.51}{E}$$

From the above relationship the force, P, in the tie beam is found to be

$$P = 61.4 \text{ kips}$$

If the maximum allowable deflection is assumed to be 10 percent of the total, 62.51/E or

$$\underline{d} = \frac{6.25''}{E}$$

the area required in the tie beam may be found from the relation

$$\underline{d} = PL/AE$$

or

$$\frac{6.25}{E} = \frac{61.4 \times 26 \times 12}{AE}$$

From which

$$A = 3{,}050 \text{ sq.in.}$$

Each plate must have sufficient steel reinforcement to prevent failure stresses induced by slab action, bending and direct stress and shearing stresses. The procedure used to obtain the steel area required for alab action and for bending plus direct stress is that used in conventional beam design. This procedure is illustrated in the following discussion. The method employed to determine the steel area required to resist tensile shearing stresses is more complicated than that required for conventional beam design and is presented in detail in the discussion which follows.

The steel reinforcing required for slab action is calculated as follows:

Slab BC: $M = 12{,}303'\text{-}\#$, $d = 7.25"$

$$A_s = \frac{M}{f_s j d} = 1.18 \text{ sq.in./ft.}$$

<u>Select #4 bars at 2" o.c.</u>

$M = 7{,}950'\text{-}\#$ at $x = 11.9'$ from B

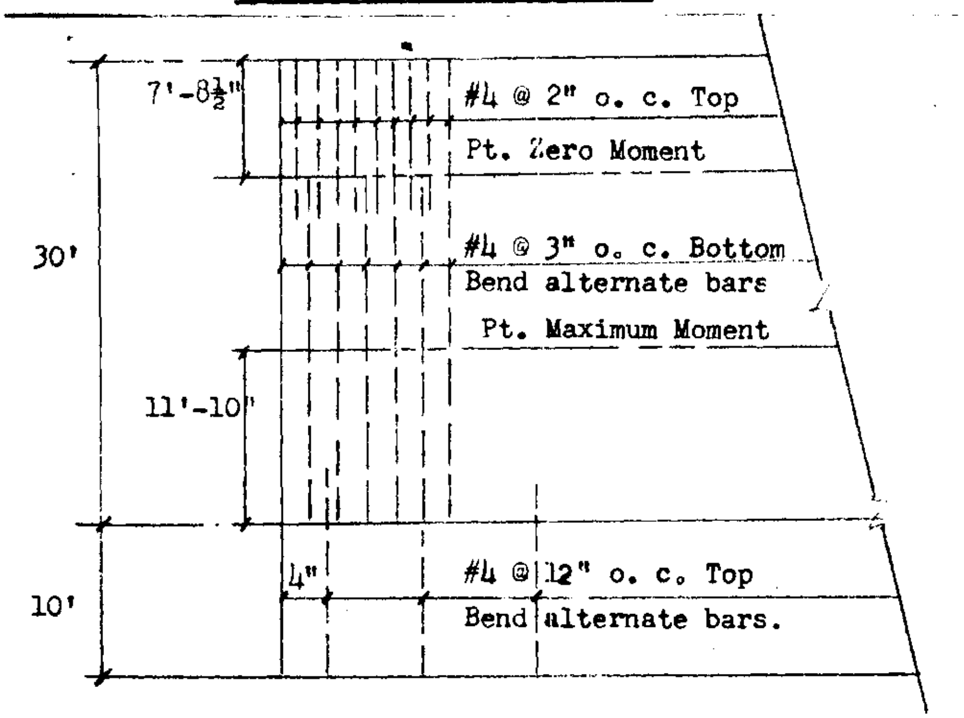
$A_s = 0.76$ sq.in./ft.

<u>Select #4 bars at 3" o.c.</u>

Slab BA: $M = 1{,}000'\text{-}\#$, $d = 7.25"$

$A_s = 0.096$ sq.in./ft.

Select #4 bars at 12" o.c.



Distribution of Slab Reinforcing

Figure 46

Figure 46 illustrates the distribution of the reinforcing bars in slab action.

The steel reinforcing required for bending plus direct load in plate action is determined as follows:

Plate 2: $M = + 210.0$ ft.-kips, $N = 110.8$ kips

Previous calculations have shown that this plate will be in compression over its entire section. The neutral axis will therefore lie outside the section; the distance, kt, will be

$$\frac{30' \times 51.4\text{psi.}}{28\text{psi.}} = 54.5\text{ft.}$$

with d' = 6 in. and t = 360. in.

Using the following equations for a column section subjected to combined bending and direct load, the required steel percentage, p, may be obtained.

$$N = \frac{f_c bt}{2k}\,(2k - 1)((1 + (n - 1)p))$$

$$\text{where } C_1 = \frac{f_c bt}{N} = \frac{2k}{(2k - 1)((1 + (n - 1)p))}$$

$$M = Ne = \frac{f_c bt^2}{12k}((1 + 12(n - 1)p(a/t)^2))$$

$$k = \tfrac{1}{2} + 1 + \frac{12(n - 1)p(a/t)^2}{12(e/t)((1 + (n - 1)p))}$$

$$\frac{d'}{t} = \frac{6}{360} = 0.0167$$

Using the preceeding equations,

$$C_1 = \frac{3000 \times 8.25 \times 360}{110800} = 0.845$$

With e = M/N = 2.1 ft.,

$$\frac{e}{t} = \frac{2.1}{30} = 0.07$$

The value, (n - 1)p is then determined as,

$$(n - 1)p = .055$$

and

$$p = 0.0061$$

or,

$$A_s = .0061 \times 8.25 \times 360 = 18.15 \text{ sq. in.}$$

The limiting spacing permitted by the A. C. I. Code is 18 inches and so,

$$\frac{360}{18} = 20 \text{ bars required}$$

The area of each bar must be at least

$$\frac{18.15}{18} = 0.930 \text{ sq. in.}$$

Select 20 - #9 bars with individual areas of 1.00 sq. in. and a combined area of 20.0 sq. in.

Plate 1: $M = 270.0$ ft.-kips, $N = 110.8$ kips

The eccentricity about the center-line of the section is

$$e' = \frac{12M}{-N} = -89.2 \text{ in.}$$

Designing, using the equivalent eccentric load method, assuming $d = 96$ in.,

$$e = e' \neq \frac{t}{2} = -31. \text{ in.}$$

Solving for F

$$F = \frac{bd^2}{12000} = 6.31 \quad \text{(Section Constant)}$$

With $E = e/12$

$$NE = -110.8 \times -2.58 = +286 \text{ ft.-kips}$$

NE is less than KF so no compression steel is required.

Solve for tensile steel area with $j = 0.939$

$$A_s = \frac{1000N}{f_s}\left(\frac{e}{jd} - 1\right)$$

$$= \frac{1000 \times -110.8}{20000}\left(\frac{-31}{90} - 1\right) = 7.46 \text{ sq. in.}$$

<u>Select 5 -#11 bars</u> to be placed symmetrically about the center of gravity of tensile resistance.

The shearing stress variation across a rectangular section, such as a beam, is parabolic. In a beam section the maximum value of the shearing stress, , at any point, y, above or below the neutral axis may be found by the equation

$$= \frac{3}{2}\frac{V}{bd^3}\left(d^2 - 4y^2\right)^{*}$$

In a rectangular simply supported beam the shearing stresses at the top and bottom of any section would therefore be zero. However, previous calculations (see Figure 41) have shown that horizontal shearing stresses exist at the junctions of the plates in this design.

To determine the magnitude of the shearing stresses in the plates

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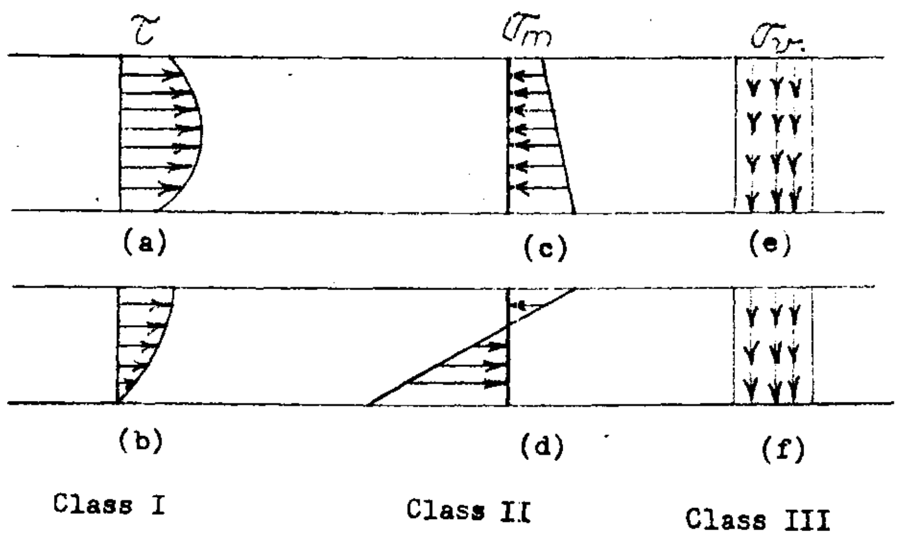
*Hool, G. and W. S. Kinne, <u>Structural Members and Connections</u>, Second Edition, New York, McGraw-Hill Book Co., Formula (4), page 27.

and the resulting steel areas required it is necessary to find the stresses acting in the plates at points along the span and also at different levels in the plates. The stresses in the plates may be classed as follows:

(1) Stresses due to the edge forces, T, plus the vertical shearing stress, v_H, due to loading conditions.

(2) Stresses due to plate bending moments, σ_m.

(3) Stresses in the vertical plane due to the load acting on the plate and the plate's own weight.



Shearing Stress in Plates

Figure 47

The shearing stresses produced by vertical shear and edge forces would have the distribution shown in Figure 47 (a) and (b) for plates 2 and 1 respectively. Taking Plate 2 as an example, the Class 1 stresses at the plate edges are 89.2 psi. (see Figure 41). A formula may be derived for determining the maximum Class 1 stress on a section in compression. This is

$$\tau = \frac{3V_o}{2th} - \frac{v_t + v_b}{4}$$

where V_o = maximum shear on section

t = width of plate

h = height of plate

v_t = shear stress at top of section

v_b = shear stress at bottom of section

The value, V_o, is determined by computing the simple beam shear at the point in question. For the extreme edge of Plate 2:

$$V_o = -4,645 \times 25 = -116,000 \text{ lbs.}$$

Then, the shearing stress at the mid-point of the plate is

$$\tau = \frac{3(-116,000)}{2 \times 8.25 \times 360} - \frac{89.2 + 89.2}{4} = -108.\text{psi.}$$

Knowing the value of stress at the top, bottom and mid-point, the stress at the one-quarter and three-quarter heights may be calculated as follows:

$$\tau_{\frac{1}{4}} = \tau_{\frac{3}{4}} = -89.2 - \frac{(108 - 89.2)}{4} = -93.9 \text{ psi.}$$

The procedure is to calculate the values of the Class I shearing stress at conventional points along the apan; in this design the stresses were determined at the eighth points along the span. These values are tabulated in Table 58.

The values of the Class II stresses may be determined from the following equation

$$\sigma_H = \frac{N}{A} \pm \frac{My}{I}$$

As both the bending moment and the longitudinal edge force, N, vary parabolically along a span with uniform load acting, the corresponding values can be determined at the eighth-points. The type of stress (tension or compression) must be considered as shown in Figure 47, (c) and (d).

The Class III stresses in the vertical plane are found from the relationship

$$\sigma_v = \frac{W}{12b}$$

where W = plate loading/foot at top plus the accumulated weight of plate/foot at the depth of plate under consideration.

Table 58. Stresses in Plate 1.

Class I

Pt. on Span	V kips	τ top	.75h	.5h	.25h	bottom
0	63.8	89.2	84.0	79.5	22.3	0
1/8	49.4	66.9	61.2	55.7	16.7	0
1/4	32.8	44.6	40.4	37.1	11.15	0
3/8	16.45	22.3	21.1	19.9	5.5	0
1/2	0	0	0	0	0	0

ClassII

Pt. on Span	M ft.-kips	N kips	σ_m top	.75h	.5h	.25h	bottom
0	0	0	0	0	0	0	0
1/8	116.	47.6	22.4	-14.0	-48.0	-83.0	-118.0
1/4	221.	91.0	41.5	-21.0	-90.0	-154.0	-225.5
3/8	251.	103.0	48.0	-24.5	-95.5	-163.0	-236.0
1/2	270.	110.8	51.0	-30.0	-109.0	-190.0	-275.0

ClassIII

Ht. of Section	W lbs.	σ_v psi.
Top	1605.	16.2
.75h	1863.	18.8
.50h	2120.	21.4
.25h	2378.	23.9
Bottom	2635.	26.6

Table 59. Stresses in Plate 2.

Class I

Pt. on Span	V kips	τ top	.75h	.5h	.25h	bottom
0	116.0	89.2	93.9	108.0	93.9	89.2
1/8	87.0	66.9	72.2	77.5	66.9	66.9
1/4	58.0	44.6	48.1	51.6	48.1	44.6
3/8	29.0	22.3	24.0	25.8	24.0	22.3
1/2	0	0	0	0	0	0

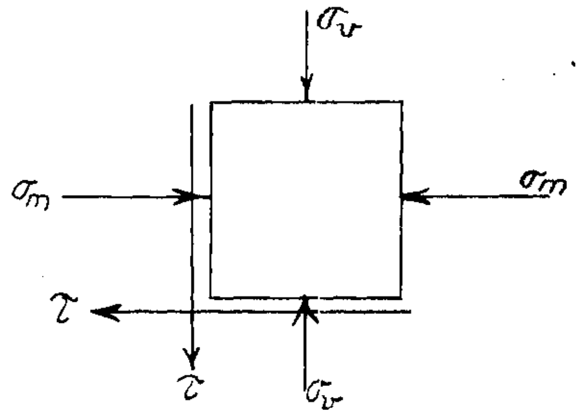
Class II

Pt. on Span	M ft.-kips	N kips	σ_m top	.75h	.5h	.25h	bottom
0	0	0	0	0	0	0	0
1/8	90.5	47.5	10.6	12.5	16.5	18.5	21.4
1/4	174.0	92.0	20.6	26.0	31.0	36.5	41.4
3/8	195.0	103.0	23.1	30.0	35.0	41.0	46.7
1/2	210.0	110.8	24.0	31.0	37.0	45.0	51.4

Class III

Ht. of Section	W lbs.	σ_v psi.
Top	2860.	28.9
.75h	3306.	33.4
.50h	3752.	37.8
.25h	4198.	42.4
Bottom	4644.	46.8

Having obtained the three classes of stress at various points and depths of plate, the stresses acting on any small element within the plate may be represented as shown in Figure 48.



Stresses on Element

Figure 48

After the stresses have been determined as illustrated in Figure 48, the magnitude and direction of the maximum principal stress may be obtained by application of the graphical procedure known as Mohr's Circle of stress. The derivation of this method will not be given here because there are many texts available which thoroughly cover the construction.[5] Instead, a typical example will be shown in Figure 49.

When the magnitude and direction of the principal stresses have been found for all points shown in Tables 58 and 59 it is possible to plot the principal stress trajectories of the two plates. As concrete is an excellent material in compression, only the principal tensile stresses need be plotted. The principal tensile stress trajectories are shown in Figure 50.

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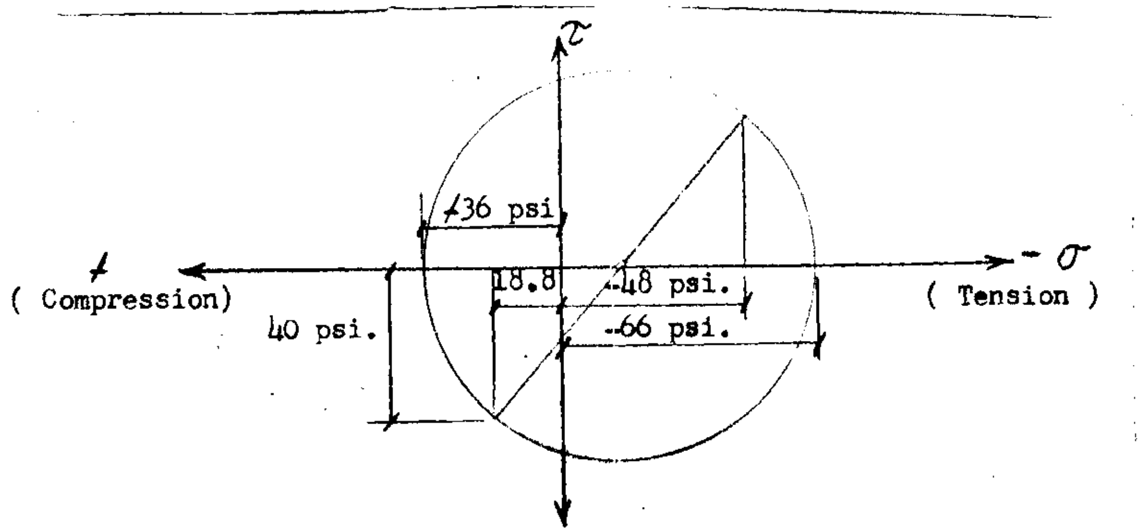
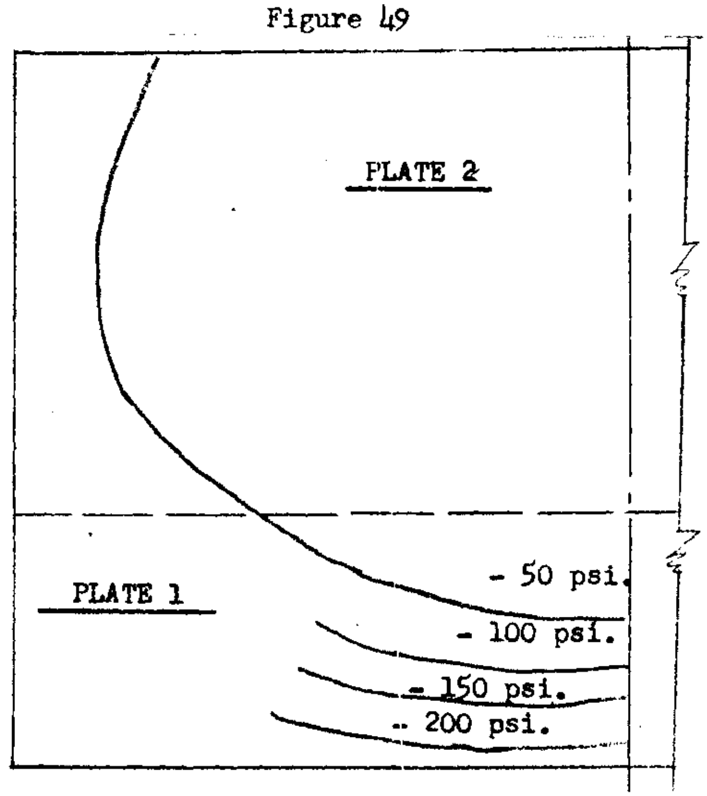
[5]See, Seely, F. B., Advanced Mechanics of Materials, First Edition, New York, John Wiley and Sons, 1950.



Principal Stresses at Mid-Height at 1/8 Point on Span - Plate 1

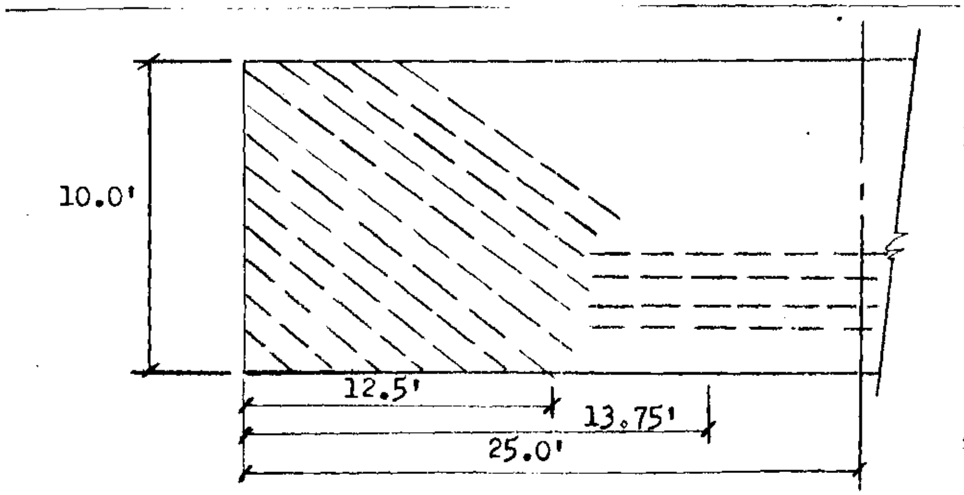
Figure 49



Lines of Principal Tensile Stress

Figure 50

To resist all diagonal tensile stresses above the allowable (90 psi.) place #3 bars on 8" centers in Plate 1 as shown in Figure 51.

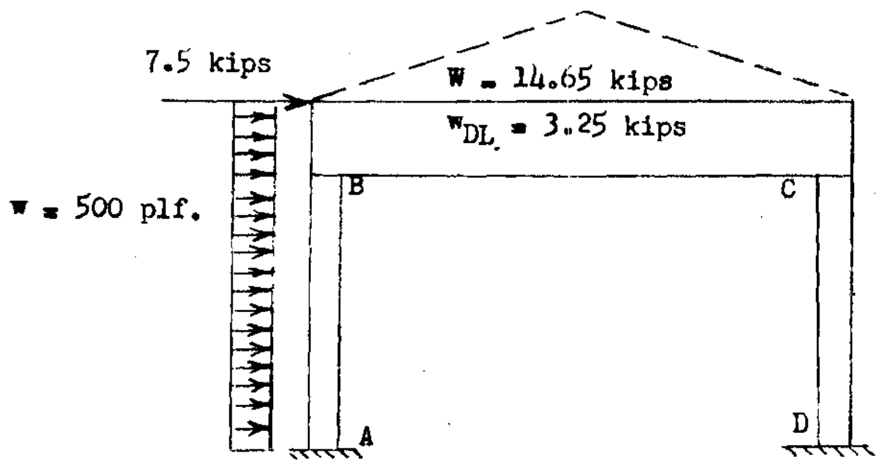


Tensile Stress Reinforcement - Plate 1

Figure 51

From preceding calculations, the minimum cross-sectional area of the tie beam was found to be 3,050.0 sq.in., with a thrust of 61.4 kips acting. If the beam is made 10 feet deep, the width, b, will then be 3050/120, or 25.4 in. Increasing the width to 26.0 in. will make the dead weight of the beam per linear foot equal to 3.25 kips. In addition to its own dead weight, the beam must carry the weight of the concrete member filling the area between the tie beam and the plates. Since this member will have no strictly structural function its thickness will be arbitrarily selected as 3 in.

The tie beam and columns will compose a rigid frame. The columns will support the weight of the entire structure and will also offer moment resistance to wind loads on the structure. The wind loading on the roof (15' x 500# = 7500#) will act horizontally at point B. Assuming the columns to be 26 in. square to conform to the width of the tie beam, the loading on the frame will be as shown in Figure 52.



Forces Acting on Rigid Frame

Figure 52

Moments of inertia for the tie beam and columns will be 181.0 ft.^{4} and 1.812 ft.^{4} respectively. The relative stiffness factors will be 57.3 and 1.0. The moments in the bent may then be determined by moment distribution as follows:

	A	B		C		D
D.L.- F.E. M.	0	0	-1790.	+1790.	0	0
W.L.- F.E. M.	-385.	+385.	0	0	0	0
Final Moment --	-357.	+441.	-441.	+ 55.	-55.	-28.

Knowing the moments, the horizontal forces at the base of the structure are

$$(357.1 - 440.8)/30 = -2.78 \text{ kips at A.}$$

A horizontal force of the same magnitude will exist at D. The forces will act toward each other. Since there is an unsymmetrical loading on the frame, it will be subject to translation. This condition, known as sidesway, is treated in all standard texts on indeterminate structures and will not be developed here; instead, only the results of calculations required to determine the resulting moments will be shown.

The previous analysis was made on the assumption that no translation of the frame took place. Now, assuming that translation occurs, an arbitrary moment of 100 ft.-kips is assumed to be applied at the ends of the vertical members. The resulting end moments, determined by moment-distribution are:

$$M_{ba},\ M_{cd} = -\ 98.81 \text{ ft.-kips}$$
$$M_{bc},\ M_{cb} = +\ 98.91$$
$$M_{ab},\ M_{dc} = -\ 99.45$$

The horizontal forces from this condition are 6.61 kips at A and D (Figure 52), both acting to the left. Because the horizontal forces at A and D from the previous distribution cancel each other, the only force remaining is the 7.5 kips at B. The moment factor is then $7.5 \div 13.32$ or 0.564. Multiplying the moments found above by this factor and adding the results to those obtained under the condition of no translation will give the true moments in the bent. These are

$$M_{ab} = -\ 413.3 \text{ ft.-kips}$$
$$M_{ba} = +\ 384.9$$
$$M_{cd} = -\ 111.2$$
$$M_{dc} = -\ \ 83.9$$

The individual members of the bent can now be designed. The girder or tie beam will be subject to bending plus axial load. The axial load will be a tensile force since it acts to keep the gable portion of the structure from spreading. At B, the eccentricity of the equivalent eccentric load is

$$E' = \frac{-385.}{-61.4} = + 6.27 \text{ ft.}$$

$$E = E' + \frac{d}{2} = 6.27 + 14.0 = 10.27 \text{ ft.}$$

The value, NE, is then

$$NE = 61.4 \times 10.27 = 630 \text{ ft.-kips}$$

The section constant, F, is

$$F = \frac{bd^2}{12,000} = \frac{26 \times (96)^2}{12,000} = 15.3$$

and

$$K_oF = 236 \times 15.3 = 3,610 \text{ ft.-kips}$$

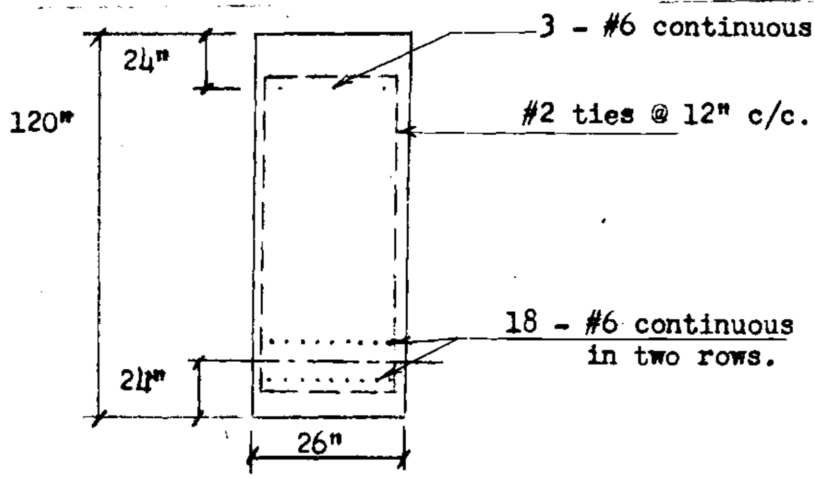
No bottom steel is required because K_oF exceeds NE. The equivalent constant, K, for the section is

$$K = \frac{NE}{F} = \frac{630}{15.3} = 42.0$$

For this value of K, the lever arm, j, ks 0.931. The top steel area is

$$A_s = \frac{1,000N}{20,000}\left(\frac{e}{jd} - 1\right) = \frac{61,400}{20,000}\left(\frac{123}{89.5} - 1\right) = 1.135 \text{ sq. in.}$$

At the center of the tie beam, by similar methods, the steel area was found to be 9.82 sq. in. at the bottom of the section. Select 3 - #6 bars (1.32 sq. in.), continuous, for the top steel and 18 - #6 bars in two rows (10.80 sq. in.) continuous, for the bottom steel. The maximum vertical shear stress is only 28.2 psi., however, the A. C. I. Code specifies that ties must be used in members subject to bending and direct stress. Select #2 tie bars at a 12 in. spacing along the span. A cross-section of the tie beam is shown in the following figure.



Steel in Tie Beam

Figure 53

The total axial load on the columns will be 50 per cent of the girder load, plus 25 per cent of the weight of the hipped plate structure, plus the column weight. The axial load is then

50% of girder load	= 91.8 kips
25% of structure load	= 21.0
column weight	= 93.0
Total Load	= 205.8 kips

For a 26 in. square column with a 20 in. spiral core

$$p = 0.01$$
$$C = 46.0$$
$$D = 6.4$$

Then

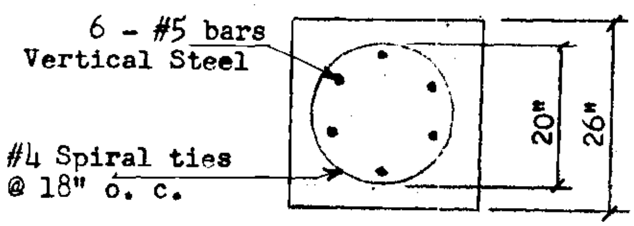
$$CDM/t = 48.7 \text{ kips}$$
$$N = 205.8$$
$$\text{Design Load} = \overline{253.5 \text{ kips}}$$

From Table 23 (Reinforced Concrete Design Handbook) the load on the concrete is 456.0 kips. The column steel is

minimum p = 0.01 bd

minimum size and number of bars = 6 - #5

select #4 spiral ties at 18 in. on centers.



Column Steel

Figure 54

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